AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 14, NUMBER 2 MARCH–APRIL, 1946

Thirty Years of X-Ray Research at the General Electric Research Laboratory

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THE General Electric Company, with its associate, the General Electric X-Ray Corporation, has had a leading part in x-ray research and development, owing largely to the contributions of W. D. Coolidge. As a close associate of Doctor Coolidge during this whole period, I consider it a special privilege to recount briefly these activities, including the most recent developments in x-ray generating equipment.

The Coolidge X-Ray Tube

Doctor Coolidge's interest in x-rays dates from Roentgen's discovery, which occurred while Coolidge was a junior student at the Massachusetts Institute of Technology. When the discovery was announced, members of the staff at the Institute immediately began experimenting with such tubes. Coolidge became so much interested that he built an electrostatic generator of his own, and with it made some of the early x-ray photographs obtained in this country. Like other pioneer investigators, he received x-ray burns which he still carries, but which, fortunately, have not been serious.

This background of interest in x-rays led inevitably to the next step. Coming to the General Electric Research Laboratory in 1905, Coolidge's first contribution was the development of ductile tungsten for lamp filaments. Once that problem was solved, his thoughts turned again to x-rays, and his first act was the introduction of the new tungsten metal as target in the x-ray tube. Shortly afterward, when Langmuir discovered the stable electron emission of tungsten filaments in high vacuum, Coolidge at once applied this technic also to x-ray tubes; so promptly, in fact, that the new tube which has since borne his name was announced almost simultaneously with Langmuir's discovery of electron space charge.[1] This was in December 1913. From that day to this, practically the whole of Coolidge's time, apart from executive duties, has been devoted to the further development of x-ray tubes and technics. Even during his directorship of the Laboratory from 1932 to 1944, his active interest in this field continued; and now, after official retirement, he is again devoting full time to his favorite subject. Though not the discoverer, he might well be called the father of the x-ray tube, by virtue of this devoted service.

X-Ray Diffraction

My own interest in x-rays began in 1915, when Sir William Bragg visited the General Electric research laboratory and told us, in his delightful manner, about the pioneer work on x-ray crystal analysis. The crucial moment came during the discussion following his lecture, when I asked if he had found the crystal structure of iron.

[1] W. D. Coolidge, "A powerful roentgen ray tube with a pure electron discharge," *Physical Rev.* 2, 409 (1913).

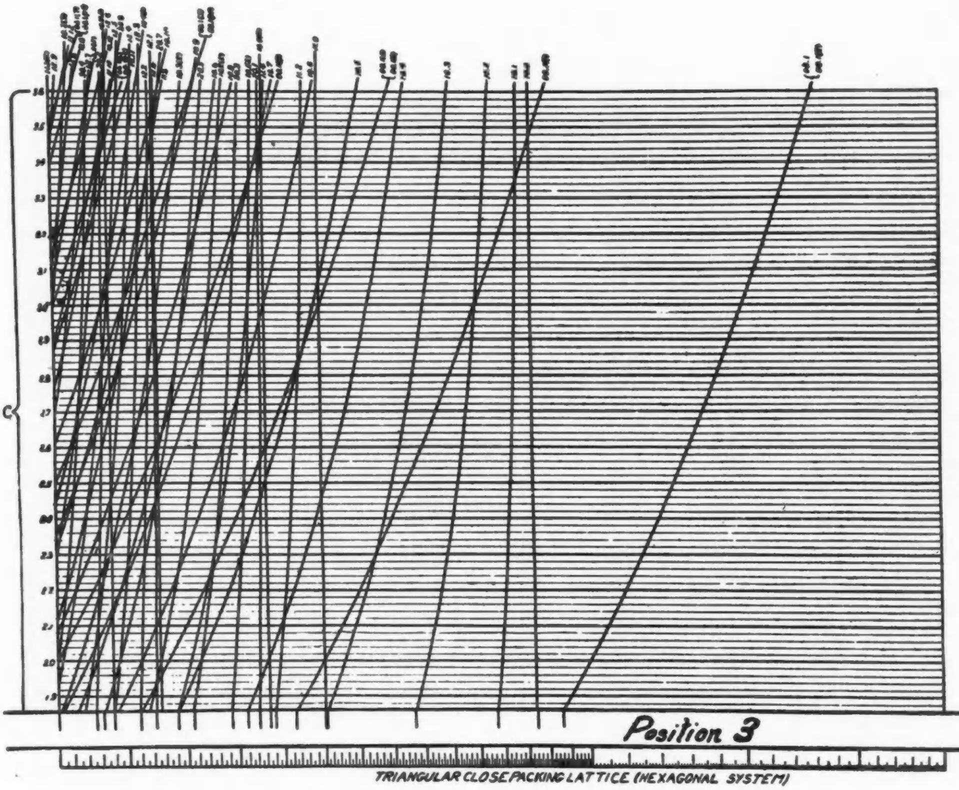

FIG. 1. Graphical analysis of crystal structure of zinc.

With characteristic modesty he replied, "No, we have tried but we haven't succeeded." That was a sufficient challenge. I decided to find the crystal structure of iron, and with it, perhaps, some evidence regarding the nature of ferromagnetism. I have often wondered whether I would have been interested in entering this field had Sir William been less modest, and, in answer to my question, had said, in effect, "Not yet, but soon."

I planned from the start to try powder diffraction, since good iron crystals were not easily obtainable. A diffraction pattern of iron filings was obtained soon. However, since I was trying to run two jobs at that time, not being willing yet to give up dynatron experiments, I did not at once check this pattern with the theoretical values for cubic crystals, a few of which had been given by the Braggs. Instead, I entrusted this task to an assistant, who reported that the pattern did not agree with any of the cubic systems. Several months later, after having had success in finding the structure of a single crystal of 3.5 percent silicon steel, and hence presumably of iron, by the Bragg method, I rechecked the original diffraction data and found that a mistake had been made; it was a perfect body-centered cubic pattern as far as could be told from the half-dozen theoretical spacings available. I report this incident in order to emphasize an important principle, namely, that a research worker should not have too many assistants—preferably not more than one.

The next step in the analysis of diffraction patterns, which proved to be quite easy, was the derivation of a matrix equation in oblique coordinates, expressing the planar spacings for any crystal system in terms of Miller indices,

axial ratios, and angles between the axes.[2] For the individual systems this equation assumed very simple forms, so that the spacings could be calculated readily for all the lines that could appear, to an order well beyond any that was likely to be observed. For cubic systems, the values for each type of symmetry were unique and, when tabulated and plotted, made possible direct comparison with any observed pattern. For tetragonal and hexagonal crystals, charts were constructed, giving the possible planar spacings as a function of axial ratio. These charts made it possible to analyze crystal structures, such as metals, for which there were no reliable data on axial ratios and symmetry. For example, zinc and cadmium had received only slight attention from crystallographers, and the axial ratios given in the literature were incorrect. Figure 1 shows the ease and certainty with which the correct structure for zinc was found by the use of the charts.[3] The procedure was to mark off the observed spacings on a strip of paper, using the same scale as that of the chart, and to move the strip about on the plot until a coincidence was found.

Figure 2 shows the equipment with which these powder diffraction patterns were obtained, allowing 15 exposures to be made simultaneously.[4] Fifty or more of these diffraction sets were manufactured by General Electric, and some are still in use. It was a good device, except that the only sealed-off tubes available for use in it were molybdenum-anode tubes, which give less dispersion and require longer exposures than anodes of lower atomic number. Attempts made at that time to produce sealed-off copper-anode tubes with thin glass windows were unsuccessful, since no stable glass could be found that was sufficiently transparent to the soft x-rays.* This defect has now been rectified by the use of beryllium windows.

These simple means sufficed for the analysis of structures of high symmetry such as metals, and more than 30 common elements, including most of the metals, were analyzed easily.[5] Some pioneer work was also done on x-ray chemical analysis, by the collection and publication of a few typical diffraction patterns[6] and demonstration of their value for determining not only the chemical composition of the sample but its physical state and crystal form as well.

I shall not try to recount the whole history of x-ray diffraction, but should like to mention especially two contributions. The first is that of Peter Debye, whose beautiful pioneer work in this field antedated mine, and is still continuing.[7] To him we owe the elegant method of analyzing crystal diffraction data that is in general use today. He also originated the analysis of amorphous substances by x-ray diffraction, and has contributed much of its theory and practice. This field is becoming of increasing importance.

The second contribution to which I would call attention is the development of chemical analysis

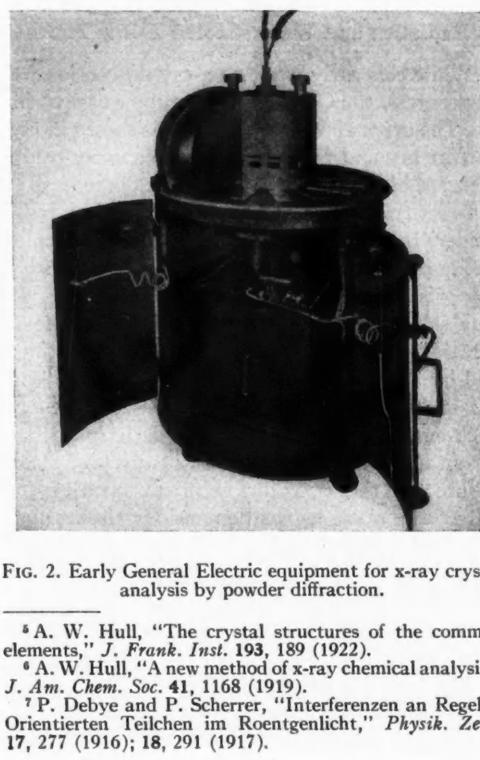

FIG. 2. Early General Electric equipment for x-ray crystal analysis by powder diffraction.

[2] A. W. Hull, "A new method of x-ray crystal analysis," *Physical Rev.* **10**, 661 (1917).

[3] A. W. Hull and W. P. Davey, "Graphical determination of hexagonal and tetragonal crystal structures from x-ray data," *Physical Rev.* **17**, 549 (1921).

[4] W. P. Davey, "A new x-ray diffraction apparatus," *Gen. Elec. Rev.* **25**, 565 (1922).

* Lindemann glass, introduced early in Europe, was not used in this country for standard tubes until 1940, because of its instability (*see* ref. 12).

[5] A. W. Hull, "The crystal structures of the common elements," *J. Frank. Inst.* **193**, 189 (1922).

[6] A. W. Hull, "A new method of x-ray chemical analysis," *J. Am. Chem. Soc.* **41**, 1168 (1919).

[7] P. Debye and P. Scherrer, "Interferenzen an Regellos Orientierten Teilchen im Roentgenlicht," *Physik. Zeits.* **17**, 277 (1916); **18**, 291 (1917).

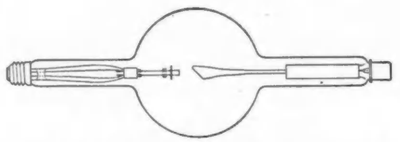

FIG. 3. First commercial Coolidge x-ray tube.

by x-ray diffraction, by I. D. Hanawalt and his associates of the Dow Chemical Company. In 1938 this group collected and published[8] data on more than 1000 standard x-ray diffraction patterns, and solicited the cooperation of other x-ray research workers. The response has been gratifying, culminating in the joint sponsorship of these tables in card form by the American Society for X-ray and Electron Diffraction, the American Society for Testing Materials, and the Institute of Physics (London). Their "Joint committee on chemical analysis by x-ray diffraction methods" has just published its index for the first supplementary set of cards, which lists data from several thousand diffraction patterns.[9]

Radiation and Water-Cooled X-Ray Tubes

While this early work on x-ray diffraction was in progress, the Coolidge x-ray tube was growing up. The original commercial type is shown in Fig. 3. War brought a need for portable x-ray equipment. This resulted in the development of a self-rectifying tube, known as the "thirty-milliampere radiator tube," having a radiation-cooled anode consisting of a heavy rod of copper with a thin disk of tungsten cast in its face (Fig. 4). The self-rectifying feature eliminated the noisy rotating-arm rectifiers which, up to this time, had been used both with the gas tubes and with the early hot-anode Coolidge tubes, to prevent back-emission. The tube was surrounded, as shown in Fig. 4, by a shield of heavy lead glass, to protect the operator, with an opening for exit of the rays. As the demand for these tubes mounted rapidly, a bottleneck was encountered in the supply of platinum for the sleeve seals. The consequent development and use of substitute alloys was one of the earliest applications of the principle of sealing thin-edged metals to glass, which had first been announced and demonstrated by W. G. Housekeeper. Later, this tube was replaced by a much smaller one, made entirely of thick lead glass, except for a window of lime glass for exit of the x-rays (Fig. 5).

The use of x-rays for therapy also increased rapidly, and called for more powerful tubes to reduce the time of treatment. A water-cooled tube was developed (Fig. 6). Its water-backed copper target, faced with thin tungsten, is capable of dissipating 15 kw. This tube was designed to operate at 250 kv on a rectified voltage supply.

The next step was a great stride forward. The x-ray tube was placed in the oil inside the high-voltage transformer. Being thus immersed in oil, the tube itself could be very small, making a compact and mobile equipment, in addition to avoiding all danger from high voltage. X-ray danger was prevented, as already noted, by making the tube itself of thick lead glass, with a thin window of leadless glass for exit of the rays. An important application of this principle is the dental radiographic set (Fig. 7) now in use by nearly every dentist in the country. The beneficial effect of this development on dentistry is hard to overestimate. It has changed x-ray dental radiography from a professional art,

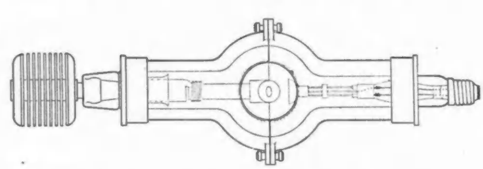

FIG. 4. Radiator-type tube, with lead-glass shield, developed for use in World War I.

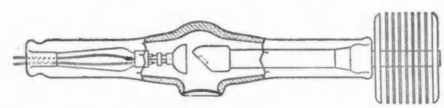

FIG. 5. Improved radiator-type tube. The tube itself is made of thick lead glass, with a window.

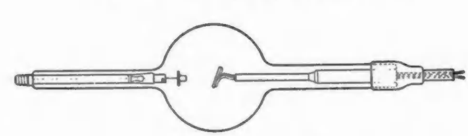

FIG. 6. Original water-cooled therapeutic tube.

[8] J. D. Hanawalt, H. W. Rinn and L. K. Frevel, "Chemical analysis by x-ray diffraction," *Ind. Eng. Chem, Anal. Ed.* **10**, 457 (1938).

[9] *Alphabetical index of x-ray diffraction patterns* (Am. Soc. Test. Mat., Aug. 1945).

costing $3.00 per picture and hence too little used, to a routine tool like the drill. Dentists, being human, were slow to give up the $3.00 prerogative, and some still cling to it, but the change will be complete soon.

Rotating-Target Tube

There was an urgent need for a high speed diagnostic tube, which would make possible instantaneous radiography of the chest, without blurring due to heart action. This was accomplished after much effort by the development of a rotating-target tube (Fig. 8). Its massive tungsten target is spun at 3500 rev/min by an induction motor, whose rotor is inside the tube, with the stator outside. The difficult problem was lubrication of the ball bearings for the rotating parts in the tube. Metallic barium, deposited in the vacuum, proved to be a satisfactory lubricant.[10] The rotation permits a six- to eightfold increase in electron current, without melting the focal spot. A further gain of threefold is obtained by using a "line focus," which, when the anode face is inclined at 20° to the direction of the x-rays, presents a square projected area. In this way a total gain of 20-fold or more is achieved,

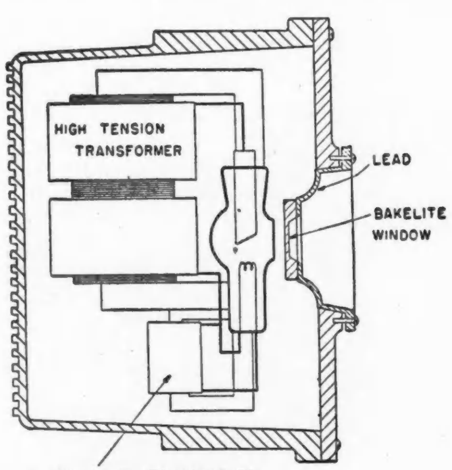

FIG. 7. Oil-immersed x-ray tube. This type is now standard for dental work.

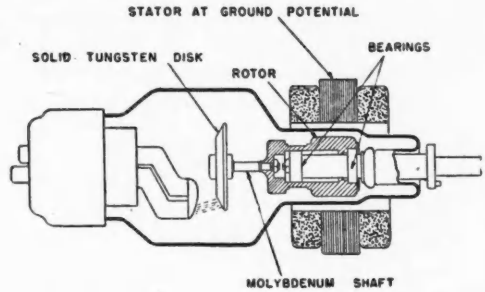

FIG. 8. Rotating tungsten target x-ray tube, for rapid radiographic work.

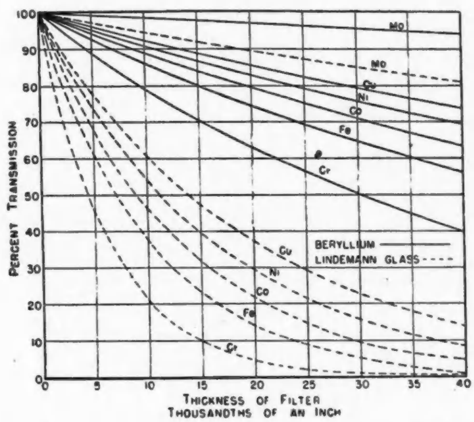

FIG. 9. X-ray transmission of beryllium and Lindemann glass for $K\alpha$ radiations of targets commonly used in diffraction work.

as compared to the standard tube with stationary target.[11]

Beryllium-Window Diffraction Tubes

The next development was of special interest to physicists. For x-ray diffraction work, a window is needed that will transmit the characteristic radiations from targets of low atomic number, namely, copper, cobalt, iron, . . ., preferably down to titanium. A special glass, consisting mainly of boron, lithium and beryllium, was developed for this purpose in Europe very early. Sealed-off tubes with windows of this "Lindemann" glass were made in Europe as early as 1933, and in this country in 1940.

[10] Z. J. Atlee, J. T. Wilson and J. C. Filmer, *J. App. Physics* **11**, 611 (1940).

[11] M. J. Gross and Z. J. Atlee, "Progress in the design of rotating anode tubes," *Am. J. Roentgenology*, **41**, 276 (1939).

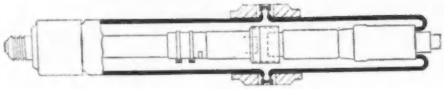

FIG. 10. Tube for x-ray diffraction, with two beryllium windows.

However, Lindemann glass is an unstable material. It also leaves much to be desired in x-ray transmission, as shown in Fig. 9. Pure beryllium is the ideal substance for such a window. The problem of making pure ductile beryllium and soldering it has finally been solved, and a special tube for diffraction work has been developed[12] (Fig. 10). It has two beryllium windows. This tube is now in standard production, and can be supplied in any of seven different target materials, namely, W, Mo, Cu, Ni, Co, Fe, Cr. In addition, tubes with targets of Ag, Cb, Zr, Ge, Mn or Ti can be furnished on special order. An oil-immersed tube with beryllium window is also available (Fig. 11). The complete oil immersion equipment, shown in Fig. 12, includes a diffraction camera mounted directly on the x-ray transformer. This arrangement is so compact that the distance from the focal spot to the first slit is less than an inch.

X-Ray Micrometer

An interesting application of the beryllium-window tube is an instrument for measuring thickness or density, developed at the General Electric Company.[13] It is adaptable to a wide range of applications. One extreme is the examination of explosive fuzes, which were checked automatically for defective filling, at the rate of 4000 per hour, during war production. At the other extreme is the measurement of thickness of paper, density of gases and concentration of chemical solutions. For example, a concentration of 0.03 percent of carbon tetrachloride vapor in air can be indicated easily. The measurement depends on absorption of the x-ray beam, and the beryllium window allows the quality of the x-rays to be adapted to the sample under inspection over a wide range of penetrating power, by adjustment of the voltage applied to the tube. The transmitted x-rays are transformed into blue light by a calcium tungstate screen, and then measured photoelectrically with an electron multiplier tube. Figure 13 shows the arrangement for controlling the thickness of sheet steel. Here two photoelectric detectors are used, and their outputs compared, thus controlling the thickness in terms of a standard sheet.

Million-Volt X-Ray Tube

For energies above 400,000 ev it is desirable to accelerate the electrons in several steps of about 100 kv each. This is accomplished by using a multisection tube, each section of which is connected to an appropriate tap on the transformer. The use of an air-cored resonance transformer makes it possible to put the tube inside the long, cylindrical secondary coil of the transformer, the primary coil being concentrated at

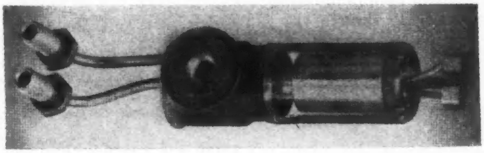

FIG. 11. Beryllium-window oil-immersed x-ray diffraction tube.

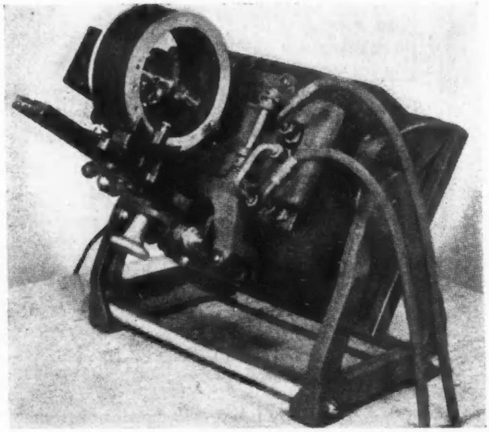

FIG. 12. Self-contained portable low voltage unit with beryllium-window tube for x-ray diffraction and microradiography.

[12] H. Brackney and Z. J. Atlee, "Beryllium windows for permanently evacuated x-ray tubes," *Rev. Sci. Instruments* **14**, 59 (1943).

[13] H. M. Smith, "X-ray inspection with phosphors and photoelectric tubes, *Gen. Elec. Rev.* **48**, 13 (1945).

the grounded end. The transformer is fed with 180-cycle power, derived from a 60-cycle source by means of a harmonic tripler. The whole is enclosed in a steel tank which provides the return circuit for the magnetic flux.

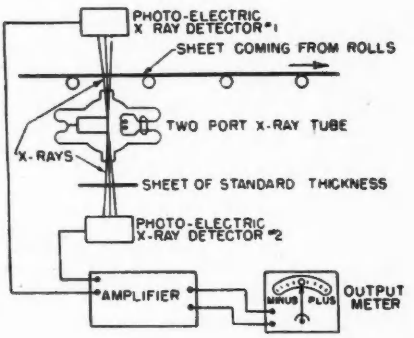

FIG. 13. Schematic circuit of x-ray thickness and density gage, arranged to control thickness of steel sheet. The x-ray, photo-tube and amplifier power supplies are not shown.

Figure 14 shows a cut-away drawing of the tube and tank. This is a second example of an x-ray tube totally enclosed in the transformer tank. In this case, Freon gas is used for insulation, rather than oil, since it is not only lighter than oil, but is actually a better insulator. A million-volt unit[14] built in this way, with a 12-section x-ray tube, weighs only 1500 lb, and is therefore mobile enough to be moved about for radiographic work. More than 50 of these tubes have been in continuous use during the war for radiography of steel structures, castings, and other war materials.

Recently a two-million-volt unit of the same type has been developed.[15] It is exactly similar to the million-volt unit, except that the tube has 24 sections, and the whole assembly is correspondingly larger. It weighs 5000 lb. It, too, is mobile, being hung from a crane and easily adjustable in any position, as may be seen in Fig. 15. The time required to radiograph an 8-in. steel casting with the two-million-volt unit is 3 min, which is only 1/100 as long as is required with the same power at a million volts.

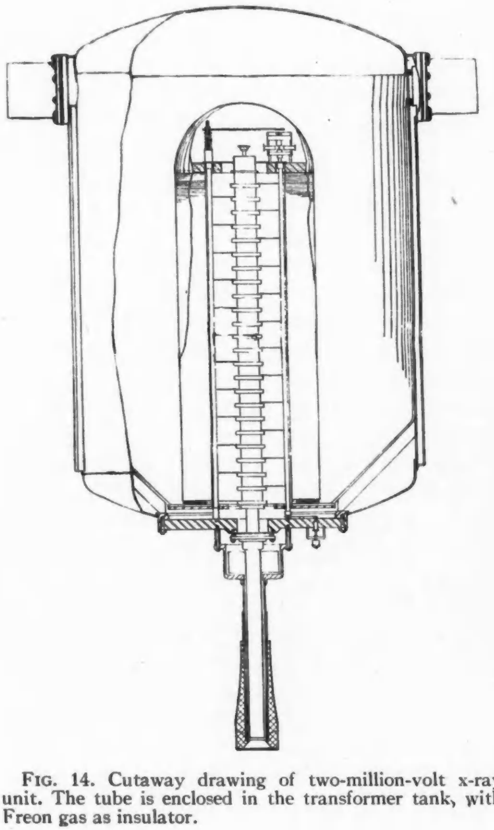

FIG. 14. Cutaway drawing of two-million-volt x-ray unit. The tube is enclosed in the transformer tank, with Freon gas as insulator.

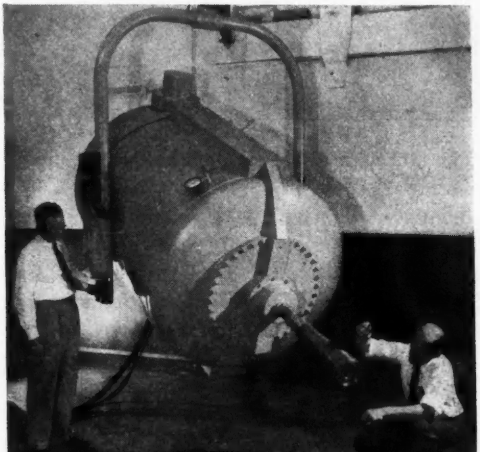

FIG. 15. Two-million-volt x-ray unit, assembled.

[14] E. E. Charlton and W. F. Westendorp, "A portable million volt outfit for industrial laboratories," *Gen. Elec. Rev.* **44**, 652 (1941).

[15] E. E. Charlton and W. F. Westendorp, "Mobile industrial x-ray unit," *Electronics* **16**, 128 (1944).

The Betatron

The newest x-ray device, the betatron, is of a novel and very simple form, in which the electron beam itself is the secondary of the 60-cycle high-voltage transformer. The idea of making such an electron induction accelerator is nearly 20 years old, but the credit for its successful accomplishment is due to D. W. Kerst, who constructed the first accelerator at the University of Illinois in 1940,[16] with the cooperation of the vacuum tube department of the General Electric X-Ray Corporation. This machine produced x-rays of energy 2 Mev. Following this development, Kerst secured a leave of absence from the University of Illinois to cooperate with the staff of the General Electric Research Laboratory on the design and construction of a larger accelerator.

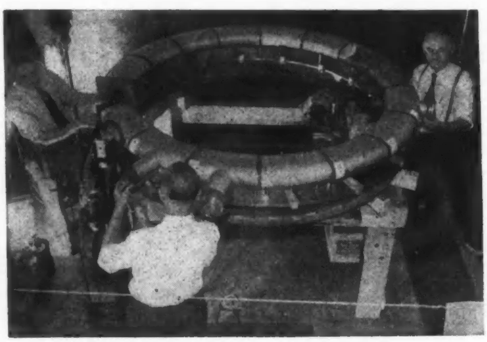

FIG. 16. Vacuum doughnut of the 100-Mev betatron.

The 20-million-volt machine resulting from this effort presented no unexpected difficulties. On completion, it was loaned to the University of Illinois. The General Electric Company then proceeded with the design and construction of a larger machine, giving 100 Mev, which was completed more than two years ago, and has been operating almost continuously since that time.[17] Figure 16 shows the glass doughnut in which the electrons circulate. It is made of 16 sections, carefully ground to the proper angle and held together by atmospheric pressure. A coat of Glyptal applied externally to the joints gives satisfactory vacuum-tightness. The inside wall has a high resistance metallic coating, high enough to avoid short circuiting the induced emf, but low enough to prevent electrostatic charging, which would cause defocusing. There are three tubulations: one for the pump, one for the electron gun that introduces the beam of electrons at about 70,000 ev, and the third for a tungsten target.

Figure 17 shows the complete transformer. Electrons are injected during a short interval of a few microseconds in each 1/60-sec. cycle, at the time when the magnetic field is passing through zero. They are then acted on continuously by an emf, equal to the rate of change of the magnetic flux through the single-turn secondary circuit, represented by their orbit in the doughnut. This emf averages 0.8 v/cm, or 400 v/rev. After a few hundred revolutions, the electrons attain a velocity practically equal to that of light. Therefore, in the accelerating period of 1/240 sec, while the magnetic flux is increasing from zero to its maximum value, they travel 1/240 of 186,000 mi, or 780 mi, making approximately 250,000 turns and attaining an energy of 100 Mev. At this energy the velocity of the electron is 99.9987 percent of the velocity of light, and its mass is 197 times that of the electron at rest.

When the electrons have attained their maximum energy, or at any earlier period if lower energies are desired, they are caused to leave the orbit by a pulse of current through an auxiliary coil, and impinge on the tungsten target, producing x-rays.

These 100-million-volt x-rays differ radically from x-rays of lower energy in one respect: they are confined to a narrow beam, parallel to the direction of the electron motion at the time of impact. This beam is only 2.0° in angular width between one-half intensity points. For radiographic work this is inconveniently narrow. Thus, a 3-ft casting would have to be placed 85 ft from the machine, in order to have the beam cover it. The penetrating power of the x-rays increases only slowly between 10 and 100 Mev, being limited principally by the production of elec-

[16] D. W. Kerst, "The acceleration of electrons by magnetic induction," *Physical Rev.* **60**, 47 (1941).

[17] W. F. Westendorp and E. E. Charlton, "A 100-million volt induction electron accelerator," *J. App. Physics* **16**, 581 (1945).

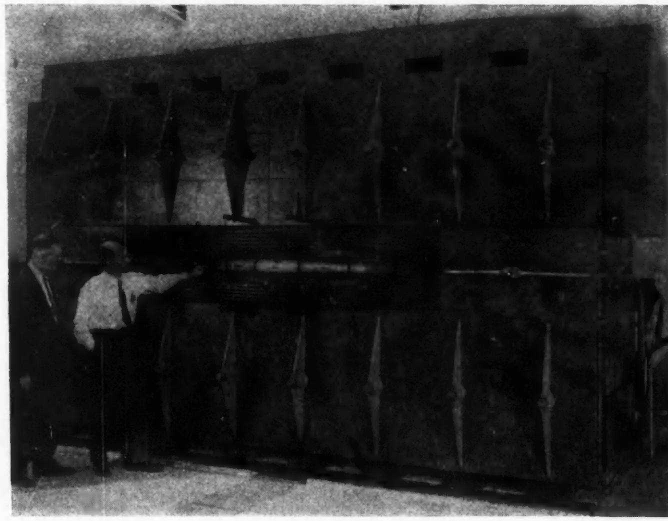

FIG. 17. The 100-Mev betatron.

tron pairs. Hence it appears that the optimum voltage for radiography may be in the neighborhood of 10–20 Mev.

A possible application of even greater potential value is x-ray therapy. Here, for the first time, we have x-rays of ample penetrating power, which retain the character of a well-defined beam in their passage through matter, by virtue of the forward scattering of the secondary x-rays.

The most interesting application of the betatron, for physicists, is the study of nuclear reactions produced by the high energy x-rays. Some of the phenomena observed in cosmic rays can be studied here under controlled conditions. For example, ionization-chamber and counter measurements can be made with the machine running continuously, at any desired voltage. For cloud-chamber work, single pulses may be used, which can be synchronized with the photographic operations. Some preliminary data of this kind have been obtained already.[18]

A natural question is, "What about still higher energies?" The 100-Mev machine weighs 130 tons. Obviously, larger machines of the same type could be built, though the weight, and hence the cost, increases as the cube of the energy. About a year ago it was pointed out by two Russian scientists, Iwanenko and Pomeranchuk,[19] that there is a limiting factor, namely loss of energy by the electrons through radiation, owing to their accelerations in the circular orbit. The maximum energy will be reached when this loss by radiation equals the gain by induction. The Russian calculations, based on classical theory and mutual independence of the radiating electrons, showed that this limit would be reached at 500 Mev, using 60-cycle induction and normal magnetic flux density. More exact calculations by a number of other physicists, taking into account the interactions of electrons, give nearly the same value. However, several methods of overcoming this limitation have been proposed recently,[20] and there is good reason for believing that electrons and x-rays of energy 1000 Mev can be produced when wanted.

[18] M. Schein, G. S. Klaiber, G. C. Baldwin and A. Hartzler, *Physical Rev.* (in press).

[19] D. Iwanenko and Pomeranchuk, *Physical Rev.* **65**, 343 (1944).

[20] E. M. McMillan, "The synchrotron, a proposed high energy particle accelerator," *Physical Rev.* **68**, 143 (1945); V. I. Veksler, "A new method for acceleration of relativistic particles." *Compt. rend. acad. sci. U.S.S.R.* **43**, No. 8; **44**, No. 9 (1944); *J. Physics U.S.S.R.* **9**, 155 (1945).

A mathematician may say anything he pleases, but a physicist must be at least partially sane.—WILLARD GIBBS.

The Scattering of X-Ray Photons

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WITH the recent celebration by the American Physical Society of the fiftieth anniversary of the discovery of x-rays, my thoughts go back to the autumn of 1920—some 25 years ago—when G. E. M. Jauncey, C. F. Hagenow and I began a series of experiments at Washington University on the scattering of x-rays. To us, x-rays were light rays of very short wavelength. We polarized them, we diffracted them, we reflected them from polished surfaces. But our chief concern was that some of the secondary rays which seemed to be scattered were more easily absorbed than the primaries from which they came. Besides, the rays of shorter wavelength were not scattered as strongly as the theories said they should be.

Those were exciting times. But if one were interested in confirming his pet theory, the work was disappointing; for, time after time, experiments gave results that were contrary to expectation. Finally, in desperation, we tried a crucial experiment. A scattered ray should at least have the same frequency and wavelength as the primary ray of which it was the echo. The test would be to compare, with a crystal x-ray spectrometer, the wavelengths of the primary and the scattered x-rays.

In those days it was no small task to obtain scattered x-rays of high enough intensity to measure their wavelength. We had to build special x-ray tubes without help of glass-blowers, for there were none in Saint Louis who knew the art, and x-ray manufacturers were not interested in our unorthodox designs. But the homemade tubes as finally built worked well enough. The x-rays from them seemed to show a definite increase in wavelength on scattering.

Fearful of personal bias, I had one of the graduate students take and record the readings of the electrometer while I shifted the crystal angles. Not knowing what we were looking for, he felt that the changing readings as we moved past one line after another were very erratic. "Too bad the apparatus wasn't working so well today," was his final comment. The data he had just taken were the ones that have since been published most frequently as showing the typical changes in wavelength of x-ray for different angles of scattering.

We knew now that x-rays were indeed increased in wavelength in the process of scattering. Was it possible that this was some kind of Doppler effect, even though the graphite block that scattered the rays was sitting stationary besides the x-ray tube? Perhaps each x-ray quantum was scattered by a single, moving electron. How much impulsive momentum would an electron receive if it scattered one quantum of x-rays? The answer came quickly:

$$mv = h/\lambda.$$

And, if the electron was moving forward with that momentum while the rays were being scattered, how much increase in wavelength would result for rays scattered at 90°? Again the answer was straightforward:

$$\lambda - \lambda_0 = h/mc = 2.4 \times 10^{-10} \text{ cm}.$$

Amazing! This was precisely the value of the wavelength change that was shown by our spectrometer measurements. It looked like a hot lead.

Then followed months of refinement of the theory and of the experiments. Apparently this Doppler effect idea was capable of describing also the relatively low intensity of scattering of x-rays of the shorter wavelengths. It was surprising how the concept of particles, or photons, simplified the theory of radiation emitted by objects moving with speeds comparable with that of light. Use of the principles of conservation of energy and of momentum as a basis for calculating the interaction of particles was a natural one, and led to the prediction of the so-called "recoil" electrons.

Actually these ideas were for the most part not new. They acquired importance because the theory became so well confirmed by experiment that it could be considered firm; and the experi-

ments, having now a theoretical basis, took on greater significance. For many years Eve and Florance and Barkla and Gray had known that scattered x-rays and gamma-rays did not behave as theory said they should. Twenty years earlier Einstein had introduced the idea of the needle-ray quantum of radiation as a basis for explaining the photoelectric effect. Indeed, while our work was under way in Saint Louis, Debye, working then at Zurich, developed independently the same theory of the scattering of x-ray photons by electrons, and was looking for someone who would put his results to the test.

Most readers today will not remember the hot debates and intensive experiments that were carried on during the next few years. First, the experiments were not believed. Then their interpretation was questioned. Better give up the principles of conservation of energy and of momentum than the beautiful simplicity of the electromagnetic wave theory of light!

The definitive experiments that showed effects of individual photons of x-rays associated with individual recoiling electrons were the final step that showed that x-rays do indeed act as particles. First, Bothe and Geiger showed by delicate tests with counters that when a recoil electron is ejected from one side of a thin foil, a scattered photon may simultaneously appear on the opposite side. A. W. Simon and I followed this with a cloud-chamber test, repeated since by several others with similar results, which showed that when a recoil electron appears, a photon is scattered in precisely the direction required by the simple theory of collision of particles.

X-rays, which by this time had shown all the wave properties exhibited by light, were thus clearly capable of acting as particles.

This result came at a most fortunate time for the development of physics. Louis de Broglie had just published his paper showing that wave groups moving in a medium of variable refractive index are equivalent to particles moving in a field of force. From this he obtained his famous expression for the wavelength of a moving particle,

$$\lambda = h/mv.$$

The fact that this expression applied perfectly to the momentum and wavelength of the x-ray photons gave life to his theory. In the hands of Schrödinger, it became the starting point of quantum wave mechanics.

Heisenberg, who had been approaching quantum mechanics through matrix theory, saw in the experimental duality of waves and particles a new method of approach, through the principle of uncertainty. This principle merely combines the limitation in determining position because of the finite resolving power associated with wavelength and the uncertainty in determining momentum because of the recoil from the emission of the particle which signals the position of the object. By a simple argument Heisenberg thus found that a limit to experimental precision was given by the expression,

$$\delta p \delta q \approx h.$$

This theory, like de Broglie's, assumed that all things have the dual properties of waves and particles. The x-ray tests showed this to be true for electromagnetic rays. Now came in quick succession the experiments of Davisson and Germer and of G. P. Thomson, showing that electrons are diffracted by crystals, and similar experiments by many others showing diffraction effects with such well-known particles as protons, hydrogen and helium atoms, hydrogen molecules and, very recently, neutrons.

A rather striking illustration of what this revolution meant in the thinking of physicists occurred when I called on J. J. Thomson in September 1927 at his home in Cambridge. He showed me with enthusiasm the photographs his son George had taken of the diffraction of cathode rays by films of aluminum and gold. Those photographs showed circular diffraction patterns closely comparable with those obtained when x-rays pass through similar films. Jauncey[1] has referred to the vigorous argument between J. J. Thomson and Lenard in the early days over the nature of cathode rays. Lenard thought of them as a type of wave; Thomson considered them to be particles. Because they could be deflected with electric and magnetic fields and because the charge on each ion could be measured, the physics world agreed that Thomson

[1] G. E. M. Jauncey, *Am. J. Physics* **13**, 362 (1945).

had proved his point; he had shown that cathode rays were particles of electricity. But here was now the old physics hero rejoicing at the achievement of his son in establishing that these same cathode rays were waves. I wonder what would have happened if George Thomson had taken his diffraction photographs at the time of the controversey between Lenard and "J. J."

Today we take it for granted that all kinds of particles have wave characteristics and that all waves have corpuscular characteristics. Which properties are predominant is only a question of the conditions under which tests are being made.

It is worth noting that the scattering of x-rays helps us in our understanding of the precise significance of particles and of waves. Briefly it is this. Whenever a ray produces a physical effect it acts as a particle; the effect is discrete and occurs in a definite position. The wave, on the other hand, serves as a convenient means of predicting where the action of the particle will probably occur. The wave is thus something of a conceptual device, while the particle has a more precise significance in terms of physical action.

When many particles are concerned in an action, for example, in the blackening of a photograph plate, the effect of the individual particles may be hidden by the action of the mass. In such cases it may be preferable to describe the action in terms of waves than of particles. An illuminating example is that of the electron structure of an atom. Instead of the older concept of electrons moving in orbits, we now think of an electron atmosphere about the nucleus whose density at any point may be expressed by the value of the appropriate Schrödinger wave function. The question sometimes arises whether in this case the electron considered as a particle continues to have any meaning. Experiments fail to determine its position or even to locate its orbit.

Experiments on the scattering of x-rays from atoms, however, show that even here the particle concept has fundamental reality. It is found that the x-rays scattered by atoms or by groups of atoms in molecules or crystals are much more intense than is to be expected if the electricity is distributed continuously within the atom. In addition to the x-rays which enter into interference relations between one atom and the next and which can correspondingly be described as "coherent" x-rays, there is an equally important part which we call, "incoherent" rays. To explain this "incoherent" radiation, we need to assume that the electricity in the atoms is not continuously distributed but consists of discrete particles whose positions are distributed at random. The wavelength of these scattered rays differs from that of coherent rays because of a kind of Doppler effect attributable to the velocity of the electron as a particle within the atom. This shows itself in the width of the modified line in the spectrum of scattered x-rays. The work of Jauncey, Dumond and others has shown that the electron velocities within the atom as thus estimated agree well with those calculated from atomic theory.

Such studies of the scattering of x-ray photons confirm the theory of the meaning of waves that Heisenberg and Bohr have emphasized. On this view, a region where a wave is of large amplitude is one for which there is a high probability of occurrence of a particle. The amplitude of a wave therefore is a measure of the probability of the presence of the particle. If, however, we are concerned with the physical actions that may occur, we must concentrate our attention on the particles themselves, for these are the sole entities that produce physical effects.

The study of the scattering of x-ray photons has likewise increased our knowledge of the characteristics of the elemental particles themselves. Consider, for example, the properties of the electron. For a long time the only characteristics of this particle revealed by experiments were its charge and its mass. Early calculations of its size turned out to have little meaning. The development of the "exclusion" principle as applied to spectra gave reason to believe that the electron likewise has a spin and should consequently have the characteristics of a tiny magnet. One of the most convincing lines of experimental evidence in this connection is the scattering of x-rays of very short wavelength. It is possible to show, following the considerations of Heisenberg's principle of uncertainty, that a particle having a mass comparable to that of an electron has a natural diameter whose

value is roughly h/mc, which in the case of an electron is equal to the wavelength of an x-ray excited by about 500 kv. If one thinks of this natural size as being the distance through which the electron charge is distributed, it is apparent that, considered from the point of view of electromagnetic theory, intereference will occur between the waves scattered from different parts of this electron. On this view, when the waves which traverse the electron have a wavelength of the same order of magnitude as the natural size, little scattering is to be expected.

If, however, the electron is spinning and has a magnetic field, the magnetic component of the traversing wave will produce torques on the electron which will change the direction of its rotation. This should give rise also to scattered x-rays. Calculation shows that for waves of the length of gamma-rays, such magnetic scattering should be more important than the well-known electrical scattering from the electron. This is perhaps the best interpretation that can be given on classical principles of the term in the formula for scattered x-rays which was added by Klein and Nishina in their quantum mechanical treatment of the problem.

Experiments of two kinds have verified this added term in the scattering formula. The first is one dealing strictly with the energy of the scattered rays. Rather refined experiments of this kind with rays of very high frequency have, as is well known, confirmed the need for the added term. The second line of evidence is considerably more specific and definite. It comes from the fact that the electric component of the rays scattered at 90° is completely polarized, while the portion that results from the magnetic dipole of the electron should be unpolarized. Shortly before the war this prediction was tested by Dr. Eric Rogers, who showed that when x-rays of very short wavelengths are scattered, they exhibit little polarization, precisely in accord with Nishina's theory.

Such results make it clear that the electron is to be considered as not only an electric charge but also a tiny magnet.

No attempt will be made here to discuss the transformation of high energy photons into positive and negative electrons. This remarkable phenomenon of pair production, revealed first by the cosmic ray experiments of C. D. Anderson and interpreted especially by J. R. Oppenheimer and his colleagues, suggests that we may be on the threshold of a new period in physics where it will be possible to understand much more thoroughly the relations among the various elemental particles. Here lies perhaps the great future of physics.

Perhaps even more important in its general human implication is the principle of uncertainty. This stems directly, as we have seen, from the experimental duality of waves and particles. In my philosophy course as a college student, I was impressed by the impossible human situation of living in a world in which we had to assume that our choices and purposes are effective while at the same time scientists seemed to have established the fact that all physical actions had been determined from the beginning of time. The problem was an old one. Lucretius in his *De Rerum Natura* asks the puzzled question, "How then are we to wrest freedom of the will from a nature in which the motions of each particle are produced by the impact of other particles upon them in an order that is fixed by natural law?" The classical expression of the verdict of the older physics is that of Laplace in the introduction to his *Celestial Mechanics:* he points out that a being with knowledge and understanding of the present state of the world, knowing the position and motion of each of its particles, would know both the past and the future as if they were the present.

Some years before the development of quantum mechanics, Schrödinger pointed out that such a statement, though it is an accurate conclusion from Newtonian mechanics, nevertheless assumes a knowledge of the reliability of our mechanical theories that goes far beyond possible experimental tests. To Bohr and Heisenberg we owe the more precise awareness of the degree of uncertainty to which our knowledge is subject.

The uncertainties to which we refer are those that necessarily become of importance in dealing with actions on a very minute scale. A single x-ray photon would be an example. We do not know at all in which direction a particular x-ray photon will be scattered when it falls upon a block of graphite. No refinement of our experi-

ments is in principle capable of making this direction predictable.

The fact that the distribution of scattered x-rays can be predicted statistically with precision is sometimes taken to mean that all large-scale events are thus predictable. But what is overlooked is that many large scale events are based at some stage upon processes on a very minute scale. Human actions are themselves examples of such events. The uncertainty in the minute event is reflected in the large scale event that follows.

A typical example is the explosion of an atomic bomb. It may be set off by the capture of a neutron that has in turn been released by some radioactive process. If, at the right moment, the initial neutron is produced, an effective explosion will occur. The type of explosion which will occur thus becomes a matter that is predictable, not precisely, but only as to its statistical probability. Here is an event of distinctly large magnitude whose occurrence is uncomfortably subject to uncertainty of the Heisenberg type.

It is considerations of this kind that have made it necessary for us to modify sharply our ideas of cause and effect. It is not appropriate here to discuss their implications with regard to the vital human problem of freedom in a world of law. It is perhaps pertinent to say, however, that no longer should a physicist try to argue, as Laplace might well have done, that effort to achieve a result can have no meaning. We are well aware now that our physical laws are not adequate for predicting a definite future, and that there is ample room within our system of physical laws to admit the effectiveness of purpose.

It was far from my thoughts when we started our experiments with x-rays 25 years ago that such work would have a bearing on such age-old problems of philosophy. We are still far from having solved these problems, but it is perhaps justifiable to feel that the way has been opened to approach a solution.

The contribution to physics of the studies of scattering of x-rays have been more concrete. They have introduced the duality of waves and particles, and have played a part in opening to physicists a new study of the relations between the elementary praticles of which the world is made.

Practical Statistics for Practical Physicists

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The purpose of this article is to present a few of the concepts and laws of statistics in the simplest possible manner, with the hope that interest in a more formal study of statistics will be aroused in the reader. No attempt to be rigorous is made in this presentation. Elegant and adequate treatments will be found listed in the bibliography in Appendix B.

Consider the measurement and classification of beta-ray spectra. One method of measuring such spectra is to measure the energies of a large sample of, say, 1000, beta-particles in a Wilson cloud chamber. Suppose the energies of a certain sample of beta-particles varied from 0.01 to 2.50 Mev. This spread of energies is an example of *variation within a sample* or within a test. There are numerous quantities or parameters, called "statistics," that define or describe different features of this variation.

Suppose that we were to take several such samples of beta-particles emanating from the same radioactive body. The samples will not be identically alike: the average energy will vary from sample to sample; so will the extrapolated end point; so will all the other properties of the samples. These variations, called sampling fluctuations, are examples of *variations among samples* or among tests, and, in the case supposed, the variations from sample to sample (or among tests) would be consistent with the variations within samples (or within tests).

On the other hand, if samples of beta-ray

energies be taken from several different radioactive substances, the variations from sample to sample would not all be consistent with the variations within the samples. Here we have the problem of distinguishing the variation within groups (that is, within the spectra themselves, not merely within the samples representing the spectra) from the variations among groups. In many cases, the differences among groups will be so large that if samples be drawn, one from each group, it will be impossible to assume that all the samples could have come from a single group; in other cases, the differences among groups may be such that the differences among samples drawn from each group may be closely like the differences to be expected to exist among the samples drawn from a single group, and powerful methods of analysis may be needed to determine whether the differences among the samples connote any real differences among the groups from which they were drawn. It is important to note that we refer to a sample of energies (or of other properties)—not to a sample of electrons—when we discuss the properties of samples.

This problem exists, on some scale or other, in every activity involving physical processes, whether the activity be mere routine testing or an elaborate research program. No matter how carefully we make a group of things, it is impossible to make all the members of the group exactly alike. This is true whether the "things" are the output of a high speed mass-production process, or the most refined measurements of the best-equipped laboratory. In either case, it is frequently necessary to be able to distinguish the variations within groups from the variations among closely neighboring groups, or to determine whether the variation from test to test in a series is consistent with the variations within the individual tests of the series. For these reasons, as well as for many others, an understanding of statistical methods and facility in the use of statistical techniques should be part of the equipment of every experimenter—physicist, chemist, engineer, or other investigator.

However, there are many competent scientists whose proficiency in statistics is not on a par with their other abilities. The following quotation[1] may explain this condition:

> Most of the mathematical theory of statistics in its present state has been developed during the past twenty years. Because of the variety of scientific fields in which statistical problems have arisen, the original contributions to this branch of applied mathematics are widely scattered in scientific literature. Most of the theory still exists only in original form.

I. DISTRIBUTION THEORY

Any homogeneous group of things—or, to be more exact, any single property or quality of the group—will be characterized by an average and the *distribution* about the average, one or more parameters, called statistics, being required to describe or to specify the distribution. Typical distributions are shown in Figs. 1–3.

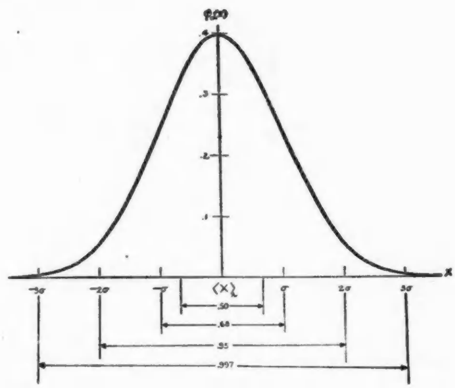

FIG. 1. The Gaussian (or Laplacian) distribution, or normal error law.

Contrary to a widespread notion, it is not necessary to assume that the distribution is "normal" in order to apply statistical methods—indeed, statistics, as such, is not concerned with the *a priori* probability that any particular distribution of physical properties will have any given form. But, once the form of some primary or parent distribution is established, statistics does show how to determine the form of secondary or related distributions.

Most physicists are aware that, under certain circumstances, the errors of measurement[2] are

[1] S. S. Wilks, *Mathematical statistics* (Princeton Univ. Press, 1943).

[2] W. E. Deming and R. T. Birge, *Rev. Mod. Physics* 6, 119 (1934).

distributed "normally," but, in many practical problems, the distribution of the actual properties is more important than the distribution of the errors of measurement; these distributions may or may not be "normal," and the observed distribution—the superposition of the distribution of the errors of measurement on the distribution of the properties—may or may not be "normal."

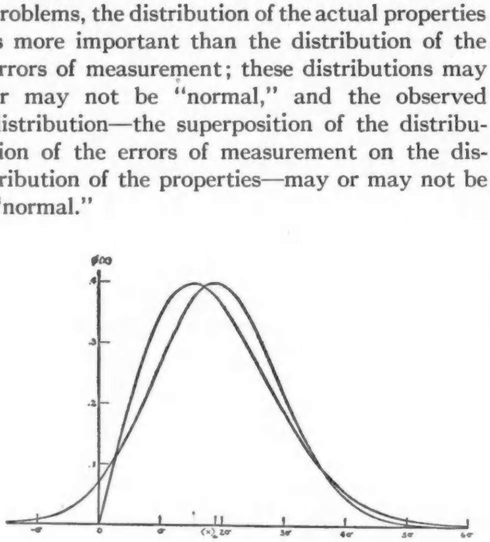

FIG. 2. One of the many familiar skew distributions, together with the "normal" distribution having the same standard deviation. Many physical properties (and manufacturing defects) are distributed according to some law, $\phi(x)=Ax^p \exp(-bx^q)$, where p is 0, 1 or 2, and q is 1 or 2. The curve shown is the graph of

$$\phi(x)=(3x/a^2)\exp(-3x^2/2a^2),$$

where $a^2=6(\langle \mathbf{x}\rangle_{\mathrm{Av}}\boldsymbol{\alpha})^2/\pi=6\boldsymbol{\sigma}^2/(4-\pi)$, or, in other words, $\langle \mathbf{x}\rangle_{\mathrm{Av}}\boldsymbol{\alpha}=1.9\boldsymbol{\sigma}$. Frequently, the eccentricities (or "out-of-roundnesses") of supposedly circular or cylindrical objects may be expected to follow this distribution. It is also the distribution of the standard deviations of samples of three specimens drawn from a normal parent universe. The skewness of this distribution is given by

$$\boldsymbol{\alpha}_3=[2(\pi-3)\pi^{\frac{1}{2}}]/(4-\pi)^{\frac{3}{2}}=0.64.$$

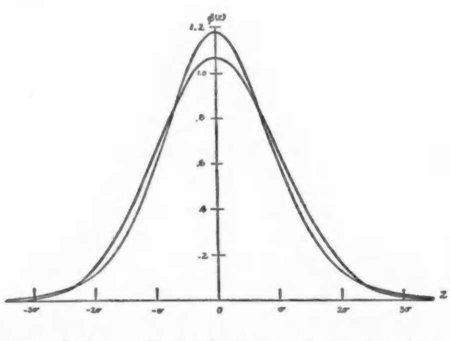

FIG. 3. A "normal" distribution, and one whose kurtosis $\boldsymbol{\alpha}_4$ is 4.2. The distribution with the greater kurtosis has the sharper peak and the longer tails. The distribution illustrated is that of "Student's" z for samples of ten specimens, Eq. (43).

1. The Population or Universe

The distribution of any particular property or quality is called a population, or *universe*. The universe includes all the potential members of the group—all the potential product of the "constant system of chance causes." Any finite group, however large, is but a sample drawn from the population or universe.

A particular property of the universe, or the "best estimate" of it obtainable from the data, will be denoted by a boldface letter; the corresponding property of a finite sample will be denoted by an italic letter, with a subscript to indicate the size of the sample.

The Distribution Function and Its Moments

Ideally, a universe is completely defined or specified by a function called the *distribution function*, or, sometimes, the density function. Thus, if $\mathrm{d}N$ be that fraction of the universe each member of which had an observed value between x and $x+\mathrm{d}x$, then

$$\mathrm{d}N=\phi(x)\mathrm{d}x, \tag{1}$$

$$\int_N \mathrm{d}N=\int_x \phi(x)\mathrm{d}x=1, \tag{2}$$

and $\phi(x)$ is the distribution function for the universe mentioned. Typical distribution functions familiar to physicists are: the Gaussian (or Laplacian) distribution, or normal error law; kinetic theory distributions of molecular velocities, energies, mean free paths, and so forth; the Planck radiation law; the Fermi distribution of the energies of beta-particles.

However, in many of the problems of everyday life, the distribution function is not obtainable from first principles. Instead, we must first draw samples from the universe. Then, from these samples, we infer properties of the distribution, called moments, and from the moments, we try to arrive at a plausible expression for the distribution function itself.

The moments of a distribution are defined as follows:

The zeroth moment, or *average*,*

* Merely to facilitate printing, we are inserting angular brackets instead of overbars to signify average.

$$\mathfrak{y}_0 \equiv \langle \mathbf{x} \rangle_{\mathrm{Av}} = \int_N x\mathrm{d}N = \int_x x\phi(x)\mathrm{d}x. \tag{3}$$

The first moment

$$\mathfrak{y}_1 = \int_N (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})\mathrm{d}N = \int_x (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})\phi(x)\mathrm{d}x = 0; \tag{4}$$

in other words, the first moment of any distribution that has any moments at all vanishes.

The second moment, or *variance*,

$$\mathfrak{y}_2 \equiv \sigma^2 = \int_N (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^2 \mathrm{d}N$$

$$= \int_x (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^2 \phi(x) dx. \tag{5}$$

The square root of the variance, or the rms variation, is called the *standard deviation*, and is denoted by σ.

The third moment

$$\mathfrak{y}_3 = \int_N (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^3 \mathrm{d}N = \int_x (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^3 \phi(x)\mathrm{d}x. \tag{6}$$

The ratio of the third moment to the cube of the standard deviation is called the *skewness*, and is denoted by α_3 (see Fig. 2). However, α_3 is not the sole measure of skewness; if a distribution is not symmetric about its average, then at least one of its odd moments, but not necessarily the third, must fail to vanish.

The fourth moment

$$\mathfrak{y}_4 = \int_N (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^4 \mathrm{d}N = \int_x (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^4 \phi(x)\mathrm{d}x. \tag{7}$$

The ratio of the fourth moment to the square of the variance is called the *kurtosis*,[3] and is denoted by α_4. It is a measure of the sharpness or flatness of the distribution at its peak (see Fig. 3). In general,

$$\mathfrak{y}_p = \int_N (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^p \mathrm{d}N = \int_x (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})^p \phi(x)\mathrm{d}x, \tag{8}$$

and

$$\alpha_p = \mathfrak{y}_p / \sigma^p. \tag{9}$$

[3] *Kurtosis* is the form used in the language and literature of statistics; *cyrtosis*, the more conventional Anglicization of the Greek word κύρτωσις, is a medical term for curvature of the spine.

TABLE I. The normal distribution: a few selected items; approximations easy to remember.

Of more interest than the ordinates of $\psi_0(z)$ are the following:

(i) $\int_{-z}^{z} \psi_0(z)\mathrm{d}z$, the area under the distribution curve between z and $-z$, or the fraction of the population or distribution lying between z and $-z$; it is the probability of obtaining a deviation of magnitude $|z|$ or less.

(ii) $1 - \int_{-z}^{z} \psi_0(z)\mathrm{d}z$, the fraction of the population outside z and $-z$, or the probability of obtaining a deviation of magnitude $|z|$ or greater; it is called simply the "probability of occurrence."

(iii) The ratio of these two probabilities, or the odds against obtaining a deviation of magnitude $|z|$ or greater.

z	$\psi_0(z)$	$\int_{-z}^{z} \psi_0(z)\mathrm{d}z$	$1 - \int_{-z}^{z} \psi_0(z)\mathrm{d}z$	$\dfrac{\int_{-z}^{z} \psi_0(z)\mathrm{d}z}{1 - \int_{-z}^{z} \psi_0(z)\mathrm{d}z}$
0	0.4	0		
1/8		1/10	9/10	
1/2		2/5	3/5	
2/3		1/2	1/2	1
1		2/3	1/3	2
1½		6/7	1/7	6½
2		0.95	1/20	20
2½		.99	1/80	80
3		.997	0.003	370
4			6×10^{-5}	16,000
5			6×10^{-7}	1.7×10^{6}

For a large variety of practical problems, it is seldom necessary to go beyond the fourth moment.

It should be noted that the α's are pure numbers, independent of the units in which x is measured.

The moments of a distribution may, at least in principle, be measured to any desired accuracy by a suitable sampling process. See Example 1, Appendix A, where the corresponding moments of a finite sample are defined. Various methods are available for inferring suitable distribution functions from a given set of moments.

When either the distribution function or the moments of a given universe have been satisfactorily determined, it is possible to compute the moments of related universes. For example, if the distribution of the diameters of steel balls being made by a particular process be determined, one can compute the moments of the distribution of the weights of the balls. Similar remarks apply to sums, differences, products, and quotients. An example familiar to physicists is that of de-

termining the distribution of molecular speeds and momentums from the previously determined distribution of the kinetic energies.

The "Normal" Distribution Function

The Gaussian (or Laplacian) distribution function, or normal error law, as it is variously called, may be denoted by $\phi_0(x)$, where

$$\phi_0(x) \equiv \frac{1}{\sigma(2\pi)^{\frac{1}{2}}} \exp\left[-(x-\langle \mathbf{x}\rangle_{\mathrm{Av}})^2/2\sigma^2\right]. \quad (10)$$

In this connection, it is convenient to change notation, as follows. Let

$$(x-\langle \mathbf{x}\rangle_{\mathrm{Av}})/\sigma = z,$$

and

$$\psi_0(z) = (2\pi)^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2}z^2\right). \quad (10')$$

Then

$$\psi_0(z) = \sigma\phi_0(x).$$

Since the "normal" distribution is symmetric about its average, all its odd moments vanish. Its even moments are:

Moments of ϕ_0	*Moments of ψ_0*
$\mu_0 = \langle \mathbf{x}\rangle_{\mathrm{Av}}$,	$\alpha_0 = 0$,
$\mu_2 = \sigma^2$,	$\alpha_2 = 1$,
$\mu_4 = 3\sigma^4$,	$\alpha_4 = 3$,
$\mu_6 = 15\sigma^6$,	$\alpha_6 = 15$,
. . . .	. . . .
$\mu_p = 1\cdot 3\cdot 5\cdots(p-1)\sigma^p$,	$\alpha_p = 1\cdot 3\cdot 5\cdots(p-1)$,

when p is even. The function $\psi_0(z)$ is given in the tables in most textbooks on statistics, in the *Handbook of Chemistry and Physics*, and in other similar collections. A brief extract from such a table is given in Table I, and a graph is shown in Fig. 1.

No physical distribution can be rigorously a "normal" universe, as this function extends to infinity in both directions. When one speaks of an actual distribution as being "normal," he means that the measurable properties of the distribution are, to some satisfactory order of accuracy, the same as the corresponding properties of a "normal" distribution. A similar statement applies to any universe whose distribution function extends to infinity in either direction.

The Gram-Charlier Series

An arbitrary distribution function can be expressed as a series of derivatives of the normal distribution function, where the coefficients of the terms in the series are linear combinations of the moments of the distribution. This series is called the Gram-Charlier series,[4] and may be written as follows:

$$\psi(z) = \sigma\phi(x) = \psi_0(z) - \alpha_3\psi_0^{\mathrm{iii}}(z)/3! + (\alpha_4 - 3)\psi_0^{\mathrm{iv}}(z)/4! - (\alpha_5 - 10\alpha_3)\psi_0^{\mathrm{v}}(z)/5! + \cdots.$$

However, as has already been mentioned, it is seldom necessary to go beyond the fourth derivative.

The reader will note a similarity between the terms of the Gram-Charlier series and the terms of the sequence of functions describing the quantum-mechanical energy states of the linear oscillator.

Pearson's Equation

A somewhat different approach was taken by Karl Pearson, who proposed the equation,

$$-\frac{\psi'(z)}{\psi(z)} = \frac{z+a}{bz^2+cz+d},$$

where the coefficients a, b, c and d are functions of the moments. The functions $\psi(z)$ fall into twelve different types, of which the "normal" distribution is an example of Type VII, and Eq. (20), below, is an example of Type III. Pearson and his followers claimed that one of the solutions to this equation is suitable for almost any problem of practical interest.[5]

Perhaps it would be well if a few words about the Gram-Charlier series, Pearson's equation, and distribution functions in general were to be included in the customary courses in mathematical methods for physicists.

2. Sampling Theory and Statistical Fluctuations

Suppose, now, we draw k samples of n members each from some population or universe. The samples will not be all alike: the averages will vary from sample to sample; so will the standard deviations; so also will all the other properties of the samples. Thus, the averages of all possible samples of size n will form a secondary population

[4] For a discussion of the Gram-Charlier series, see A. Fisher, *Mathematical theory of probability* (Macmillan, 1922).

[5] For an interesting account of the work of Karl Pearson, see T. L. Kelly, *Statistical methods* (Macmillan, 1923).

of universe; their variances will form another; each of the higher moments and other properties will form still other secondary universes.

The moments of these secondary universes will be related to those of the parent universe in definite ways. Some of these relations are quite easy to derive; others are quite tedious to obtain; still others remain to be found.

The Universe of Averages

All of the moments of the universe of averages of samples of a given size n are expressible in terms of the moments of the parent universe.

The average $\langle \mathbf{x}_n \rangle_{\mathrm{Av}}$ of this universe of averages is identically the same as that of the parent universe itself, regardless of the form of its distribution. One can therefore estimate the average of any population as closely as he pleases by taking a sufficiently large sample.

To see this, let the k samples, of n specimens each, be considered as one large sample of size kn. The grand average of the k averages will be the same as that of the kn specimens. As k increases indefinitely, this average approaches that of the parent universe. Thus,

$$\langle \mathbf{x}_n \rangle_{\mathrm{Av}} = \lim_{k\to\infty} \sum \bar{x}_n/k = \lim_{k\to\infty} \sum\sum x/kn = \langle \mathbf{x} \rangle_{\mathrm{Av}},$$

or, in terms of the parent distribution function,

$$\langle \mathbf{x}_n \rangle_{\mathrm{Av}} = \int \bar{x}_n \phi(x)\mathrm{d}x = \int (\sum x/n)\phi(x)\mathrm{d}x$$

$$= (1/n)\sum \int x\phi(x)\mathrm{d}x = \langle \mathbf{x} \rangle_{\mathrm{Av}}. \quad (11)$$

The derivations of Eqs. (12), (16), etc., below, are somewhat similar to that of Eq. (11), but they are not quite so simple.

The variance of the universe of averages is

$$\sigma_{\bar{x}_n}{}^2 = \sigma^2/n; \quad (12)$$

or the standard deviation of the mean is

$$\sigma_{\bar{x}_n} = \sigma/n^{\frac{1}{2}}. \quad (12')$$

These results are independent of the form of $\phi(x)$ of the parent universe.

Equations (11) and (12′) are the common-sense relations used instinctively even by those who have had no formal training in statistics. They are the quantitative statements of the common notion that the reliability of an estimate is proportional to the square root of the number of observations upon which it is based.

For the skewness and kurtosis of the universe of averages of samples of size n, we have

$$\alpha 3_{\bar{x}(n)} = \alpha_3/n^{\frac{1}{2}}, \quad (13)$$

$$\alpha 4_{\bar{x}(n)} = [(\alpha_4 - 3)/n] + 3. \quad (14)$$

Equations (13) and (14) tell us that no matter what the form of $\phi(x)$ of the parent universe, the larger the sample size, the closer is the universe of averages to a "normal" universe. If the parent universe be normal to begin with, the universe of averages of samples of any size is normal, and the distribution function is

$$\phi(\bar{x}_n) = \frac{n^{\frac{1}{2}}}{\sigma(2\pi)^{\frac{1}{2}}} \exp[-n(\bar{x}_n - \langle \mathbf{x} \rangle_{\mathrm{Av}})^2/2\sigma^2],$$
$$\psi(\bar{z}_n) = n^{\frac{1}{2}}(2\pi)^{-\frac{1}{2}} \exp[-n(\bar{z}_n)^2/2], \quad (15)$$

where

$$\bar{z}_n = \sum (x - \langle \mathbf{x} \rangle_{\mathrm{Av}})/n\sigma = (\bar{x}_n - \langle \mathbf{x} \rangle_{\mathrm{Av}})/\sigma = \sum z/n.$$

Thus, as far as averages are concerned, one can use the "normal" law as a very close approximation of the results to be expected in any sampling scheme, even for samples as small as 5 or 10. In any event, one can always use Eq. (12) or (12′) to determine whether the variation from sample to sample is consistent with the variation within samples.

The Universe of Variances

The moments of the universe of variances of samples of a given size, drawn from a given parent universe, are also expressible in terms of the moments of the parent universe. Here, again, the average of this universe is independent of the form of $\phi(x)$:

$$\langle \sigma_n{}^2 \rangle_{\mathrm{Av}} = (n-1)\sigma^2/n, \quad (16)$$

or the average of the variances of all samples obtainable from a given parent universe is always less than the variance of the parent universe itself. Thus, for the variance of the parent universe, the "best estimate" obtainable from k samples of size n is

$$\boldsymbol{\sigma}^2 = [n/(n-1)]\langle\sigma_n^2\rangle_{\text{Av}} = n\sum \sigma_n^2/k(n-1)$$
$$= \sum\sum (x-\bar{x}_n)^2/k(n-1)$$
$$= [\sum\sum x^2 - n\sum (\bar{x}_n)^2]/k(n-1). \quad (16')$$

If the samples are not all of the same size n, the "best estimate" of the variance of the parent universe is

$$\boldsymbol{\sigma}^2 = (\sum n\sigma_n^2)/(\sum n-k)$$
$$= \sum\sum (x-\bar{x}_n)^2/(\sum n-k)$$
$$= [\sum\sum x^2 - \sum n(\bar{x}_n)^2]/(\sum n-k). \quad (16'')$$

In such an expression the numerator is called "the sum of squares," and the denominator is called "the number of degrees of freedom" of the variance being computed.

The higher moments of the universe of variances of samples of size n are lengthy expressions —for example, the variance of this universe involves both the variance and the kurtosis of the parent universe. If the parent universe be "normal," the higher moments are

$$\boldsymbol{\sigma}_{\sigma(n)^2}{}^2 = 2(n-1)\boldsymbol{\sigma}^4/n^2, \quad (17)$$

$$\left.\begin{aligned} \boldsymbol{\mu}3_{\sigma(n)^2} &= 8(n-1)\boldsymbol{\sigma}^6/n^3, \\ \boldsymbol{\alpha}3_{\sigma(n)^2} &= 4/[2(n-1)]^{\frac{1}{2}}, \end{aligned}\right\} \quad (18)$$

$$\left.\begin{aligned} \boldsymbol{\mu}4_{\sigma(n)^2} &= 12(n-1)(n+3)\boldsymbol{\sigma}^8/n^4, \\ \boldsymbol{\alpha}4_{\sigma(n)^2} &= 3(n+3)/(n-1). \end{aligned}\right\} \quad (19)$$

In 1908, Gossett[6] observed that these moments were the same as those of the function

$$\phi(\sigma_n^2) = \frac{n^{(n-1)/2}\sigma_n^{n-3}\exp(-n\sigma_n^2/2\boldsymbol{\sigma}^2)}{(2\boldsymbol{\sigma}^2)^{(n-1)/2}\Gamma\left(\dfrac{n-1}{2}\right)}. \quad (20)$$

He also showed that it was plausible to assume that this was the actual distribution function for the universe of variances. This assumption was later verified rigorously by R. A. Fisher[7] and by Karl Pearson[8] in 1915. The same equation had also been derived by Helmert[9] in 1876, but by 1900 his work had long been forgotten.

The Universe of Standard Deviations

Having found either the distribution function or the moments of the universe of variances of samples of size n drawn from a given universe, one can then find those of the universe of standard deviations of the same samples. If the parent universe be "normal," the desired distribution function, its average and its variance are:

$$\phi(\sigma_n) = \frac{n^{(n-1)/2}\sigma_n^{n-2}\exp(-n\sigma_n^2/2\boldsymbol{\sigma}^2)}{2^{(n-3)/2}\boldsymbol{\sigma}^{n-1}\Gamma\left(\dfrac{n-1}{2}\right)}; \quad (21)$$

$$\langle\boldsymbol{\sigma}_n\rangle_{\text{Av}} = \sqrt{2}\,\Gamma(n/2)\boldsymbol{\sigma} \Big/ n^{\frac{1}{2}}\Gamma\left(\frac{n-1}{2}\right) = c_2\boldsymbol{\sigma}; \quad (22)$$

$$\boldsymbol{\sigma}_{\sigma_n}{}^2 = \langle\boldsymbol{\sigma}_n^2\rangle_{\text{Av}} - (\langle\boldsymbol{\sigma}_n\rangle_{\text{Av}})^2 = [(n-1)/n - c_2^2]\boldsymbol{\sigma}^2; \quad (23)$$
$$\rightarrow \boldsymbol{\sigma}^2/2n, \quad n \geqslant 5;$$
$$\boldsymbol{\sigma}_{\sigma_n} \rightarrow \boldsymbol{\sigma}/(2n)^{\frac{1}{2}}, \quad n \geqslant 5. \quad (23')$$

Graphs of $\phi(\sigma_n)$ for $n=4$ and for $n=10$, together with a table for various values of n, were given by Gossett.[6] A graph of $\phi(\sigma_n)$ for $n=3$ is shown in Fig. 2. The coefficient of $\boldsymbol{\sigma}$ in Eq. (22) is denoted by c_2 in the literature on industrial statistics and quality control; a few typical values are given in Table II. Although Eq. (23) applies rigorously only to samples drawn from a "normal" population, Eq. (23′) is a suitable approximation for most practical purposes if n is equal to 5 or more.

The Universe of Skewnesses

For large samples (say, $n=200$ or more), the following relations are satisfactory:

TABLE II. Properties of samples drawn from a "normal" universe. The columns headed $\boldsymbol{\sigma}_{\bar{x}_n}$, $\langle\boldsymbol{\sigma}_n\rangle_{\text{Av}}$, $\boldsymbol{\sigma}_{\sigma_n}$, $\langle\mathbf{R}_n\rangle_{\text{Av}}$, and $\boldsymbol{\sigma}_{R_n}$ give the ratios between these quantities and the standard deviation of the parent universe. The columns headed $\langle\boldsymbol{\sigma}_n^2\rangle_{\text{Av}}$ and $\boldsymbol{\sigma}_{\sigma(n)^2}$ give the ratios between these quantities and the variance of the parent universe.

Size of sample, n	Averages, $\boldsymbol{\sigma}_{\bar{x}_n}$	Standard deviations $\langle\boldsymbol{\sigma}_n\rangle_{\text{Av}}$	Standard deviations $\boldsymbol{\sigma}_{\sigma_n}$	Variances $\langle\boldsymbol{\sigma}_n^2\rangle_{\text{Av}}$	Variances $\boldsymbol{\sigma}_{\sigma(n)^2}$	Ranges $\langle\mathbf{R}_n\rangle_{\text{Av}}$	Ranges $\boldsymbol{\sigma}_{R_n}$
5	0.45	0.84	0.32	0.80	0.32	2.33	0.86
10	.32	.92	.22	.90	.18	3.08	.80
15	.26	.95	.19	.93	.12	3.47	.75
20	.22	.96	.16	.95	.09	3.74	.73
25	.20	.97	.14	.96	.08	3.93	.70
50	.14	.98	.10	.98	.04	4.5	.65
100	.10	.99	.07	.99	.02	5.0	.60
200	.07	.997	.05	.995	.01	5.5	.57
500	.04		.03	.998	.004	6.0	.52
1000	.03		.02	.999	.002	6.5	.50

[6] "Student," *Biometrika* **6**, 1 (1908). W. S. Gossett, to whom the theory of sampling owes much, made his contributions under the name of "Student."
[7] R. A. Fisher, *Biometrika* **10**, 507 (1915); *Proc. Camb. Phil. Soc.* **21**, 655 (1923).
[8] K. Pearson, *Biometrika* **10**, 522 (1915).
[9] F. R. Helmert, *Astron. Nachrichten* **88**, 122 (1876). This paper was brought to light by K. Pearson, *Biometrika* **23**, 416 (1932).

$$\langle \alpha 3_n \rangle_{Av} = \alpha_3; \tag{24}$$

$$\sigma_{\alpha 3(n)}{}^2 = 6/n \quad \text{or} \quad \sigma_{\alpha 3(n)} = (6/n)^{\frac{1}{2}}. \tag{25}$$

Thus, large samples are required for the estimation of α_3—12 times as large as those required for estimating σ with the same accuracy.

The Universe of Kurtoses

For large samples ($n = 500$ or more), we have

$$\langle \alpha 4_n \rangle_{Av} = \alpha_4; \tag{26}$$

$$\sigma_{\alpha 4(n)}{}^2 = 24/n \quad \text{or} \quad \sigma_{\alpha 4(n)} = (24/n)^{\frac{1}{2}}. \tag{27}$$

Thus, to estimate α_4 with the same precision as σ, we must use samples at least 50 times as large.

For samples drawn from a "normal" parent universe, the distribution of kurtoses is somewhat skewed, with the longer tail in the direction of large kurtoses.

The Universe of Ranges

The range R_n, or the extreme variation of the sample, is the convenient statistic to use for certain purposes when one has a large number ($k > 10$) of small samples ($n \leq 10$). Unlike the secondary universes just described, the universe of ranges becomes less and less "normal" as n increases. Its distribution function may be written

$$\phi(R_n) = n(n-1)\int_{-\infty}^{\infty - R_n}\left[\int_x^{x+R_n}\phi(x)dx\right]^{n-2} \times \phi(x)\phi(x+R_n)dx. \tag{28}$$

However, except for a few special cases, it has not been found possible to solve this equation for $\phi(R_n)$. In 1925, Tippett[10] found the numerical values of the moments of $\phi(R_n)$ for samples of various sizes drawn from a "normal" universe. In the literature of quality control, the ratio between the average range of samples of given size n and the standard deviation of the parent universe is denoted by d_2, so that

$$\langle \mathrm{R}_n \rangle_{Av} = d_2 \sigma. \tag{29}$$

A few typical values of d_2 are given in Table II.

Thus, if one has many small samples, he can estimate the standard deviation of the parent universe from Table II and Eq. (29). The estimate thus obtained is good enough for many purposes, and the method can be used by untrained personnel.

[10] L. H. C. Tippett, *Biometrika* **17**, 364 (1925).

The Differences Between Two Independent Samples

Frequently, we have only two samples, and are confronted with the question: may these two samples be reasonably considered to have come from the same population or universe, or, what amounts to the same thing, from parent universes having the same average, variance, and so forth. To answer this question, it is convenient to resolve it into successive parts: (i) could the parent universes have had the same average?; (ii) could they have had the same standard deviation?; and so on. To answer these questions, we compare the observed differences between the two samples with the differences expected to be found to exist between two samples drawn from a single population or universe.

To answer the first question, we have to know something about the distribution of two ratios, denoted by u and t. The choice between which ratio to use depends upon the state of our knowledge of the standard deviation of the universe (or universes) from which the samples are presumed to have been drawn.

We have already mentioned that the distribution of $\bar{x}_n$ is approximately normal, with standard deviation $\sigma/n^{\frac{1}{2}}$. Consider now the distribution of the differences between the averages of all possible pairs of samples, one having m specimens, the other having n (m and n need not be equal), obtainable from a given universe. This distribution is also nearly "normal." Its average is zero. Its variance is

$$\sigma_{\bar{x}_m - \bar{x}_n}{}^2 = \sigma_{\bar{x}_m}{}^2 + \sigma_{\bar{x}_n}{}^2 = \sigma^2(1/m + 1/n). \tag{30}$$

The average value of $|\bar{x}_m - \bar{x}_n|$ is $(2/\pi)^{\frac{1}{2}}\sigma_{\bar{x}_m - \bar{x}_n}$, or $0.8\sigma[(m+n)/mn]^{\frac{1}{2}}$. Thus, if the two samples be of the same size ($m = n$), the average difference between the two averages should be $1.12\sigma/n^{\frac{1}{2}}$. However, it should be less than this about two-thirds of the time; for only one pair in 20 should it be twice this, and for only one pair in 370 should it be three times this.

The ratio, $(\bar{x}_m - \bar{x}_n)/\sigma_{\bar{x}_m - \bar{x}_n}$ is denoted by u, where

$$\phi(u) = \psi_0(u) = (2\pi \exp[u^2])^{-\frac{1}{2}}. \tag{31}$$

This ratio is useful when σ is known (or can be assumed) from a wealth of data independent of the two samples. In a large variety of problems, this is frequently the case: in the manufacture of many products, for example, the standard deviations of most properties remain quite constant, whereas the averages might vary from machine to machine, or from day to day, and so on. In such cases, the object of the test is to determine whether the averages actually do vary in an undesirable manner or not.

However, if the only estimate of σ is that obtained from the two samples themselves, a different procedure is required.

We have already seen that the best estimate of σ^2 obtainable from only two samples is, from Eq. (16″),

$$\sigma^2 = (m\sigma_m{}^2 + n\sigma_n{}^2)/(m+n-2).$$

Combining this with Eq. (30), we obtain as the best estimate of $\sigma_{\bar{x}_m-\bar{x}_n}$,

$$\sigma_{\bar{x}_m-\bar{x}_n} = [(m\sigma_n{}^2 + n\sigma_m{}^2)(m+n)/(m+n-2)]^{\frac{1}{2}}. \quad (32)$$

We then form the ratio

$$t = (\bar{x}_m - \bar{x}_n)/[(m\sigma_n{}^2 + n\sigma_m{}^2) \times (m+n)/(m+n-2)]^{\frac{1}{2}}. \quad (33)$$

This ratio is known as "Student's" t. For samples drawn from a "normal" parent universe, Gossett[6] obtained:

$$\phi(t) = \frac{\Gamma\left(\dfrac{m+n-1}{2}\right)\left(1+\dfrac{t^2}{m+n-2}\right)^{-(m+n-1)/2}}{\Gamma\left(\dfrac{m+n-2}{2}\right)[\pi(m+n-2)]^{\frac{1}{2}}}; \quad (34)$$

$$\langle \mathbf{t} \rangle_{\mathrm{Av}} = 0; \quad (35)$$

$$\sigma_t{}^2 = 1/(m+n-4), \quad (m+n) > 4; \quad (36)$$

$$\alpha_{3t} = 0; \quad (37)$$

$$\alpha_{4t} = 3 + 6/(m+n-6), \quad (m+n) > 6. \quad (38)$$

Thus, for small samples, the kurtosis of the distribution of t is greater than that of u, in agreement with the fact that, for samples of a given size, smaller standard deviations are more frequent than the larger ones. This skewness in the distribution of σ_n has the effect of elongating the tails of the distribution of t, a fact that is frequently ignored by those not trained in statistical methods.

For large samples,

$$\lim_{m+n\to\infty} t = u, \quad \lim_{m+n\to\infty} \phi(t) = \psi_0(u). \quad (39)$$

As a practical matter, however, we can substitute u for t and $\psi_0(u)$ for $\phi(t)$ if m and n are each larger than 30, but not otherwise.

Table III is a short table of values of $\phi(t)$ for samples of 5, 10, 15 and 20 specimens.

The usefulness of "Student's" t is enhanced by the fact that, since the distribution of $(\bar{x}_m - \bar{x}_n)$ is always nearly "normal," $\phi(t)$ is but little affected by the properties of $\phi(x)$ of the parent universe. This is not true for the corresponding function for the differences between the standard deviations of two samples drawn from a single universe, so that the problem of answering the second question, "Could the parent universes have had the same standard deviation?," is more difficult.

On the other hand, the chief object of a test might be to determine whether the material made by one process is actually more uniform than similar material made by another process.

In many cases, we can start with the approximation stated in Eq. (23′). Then the standard

Table III. "Student's" t. This table illustrates a second common method of tabulating statistical functions. Here we have the values of the argument corresponding to given probabilities of occurrence, instead of the probability of occurrence of given values of the argument, as in Table I.

"Degrees of freedom"	Probability of occurrence 0.9	0.5	0.1	0.05	0.01
8	0.130	0.706	1.86	2.31	3.36
13	.128	.694	1.77	2.16	3.01
18	.127	.688	1.73	2.10	2.88
23	.127	.685	1.71	2.07	2.81
28	.127	.683	1.70	2.05	2.76
33	.127	.683	1.69	2.03	2.73
38	.127	.683	1.68	2.02	2.71
∞	0.1257	0.6745	1.645	1.960	2.576

For example, one tenth of the pairs of samples of five specimens each (eight "degrees of freedom"), obtainable from a "normal" parent universe, will differ from each other in such a way that t for the pair will be 1.86 or larger; another tenth of such pairs will differ in such a way that t will be 0.130 or less. An "infinite number of degrees of freedom" corresponds to the normal distribution itself; the figures given on this line are the more exact values for some of the approximations given in Table I.

deviation of the difference between the standard deviations of pairs is

$$\sigma_{\sigma_m-\sigma_n}=(\sigma_{\sigma_m}{}^2+\sigma_{\sigma_n}{}^2)^{\frac{1}{2}}=\sigma(1/2m+1/2n)^{\frac{1}{2}}, \quad (40)$$

and the best estimate of σ is that given by Eq. (16″), as before. We now form the ratio,

$$\tau=(\sigma_m-\sigma_n)/[(m\sigma_n{}^2+n\sigma_m{}^2)\times(m+n)/2(m+n-2)]^{\frac{1}{2}}. \quad (41)$$

Although the distribution of τ is not known for many parent universes, the following two statements hold for most problems of practical interest, if each sample contains at least ten specimens: (i) for not more than one pair of samples in 20 should $|\tau|$ be greater than $2\frac{1}{4}$; (ii) for not more than one pair in 100 should $|\tau|$ exceed 3.

Even if the value of τ indicates that the two parent universes did not have the same standard deviation, one can still use "Student's" t to determine whether they could have had the same average.

"Student's" z

The object of many tests is to determine whether a lot of material conforms to some contract or specification. This is equivalent to determining whether a given sample can be considered to have come from a population of given $\langle \mathbf{x}\rangle_{\text{Av}}$ and σ, or, frequently, of given $\langle \mathbf{x}\rangle_{\text{Av}}$ alone. To answer the first question, we have merely to form the ratio

$$\bar{z}_n=(\bar{x}_n-\langle \mathbf{x}\rangle_{\text{Av}})/\sigma=\sum z/n$$

and apply the "normal" distribution law, Eq. (10′), for averages of samples of size n, according to which:

(i) The average value of $\bar{z}_n$ is equal to 0 (or, in other words, $\langle \mathbf{z}_n\rangle_{\text{Av}}=0$);

(ii) The average value of $|\bar{z}_n|$ is equal to $(2/n\pi)^{\frac{1}{2}}=0.8/\sqrt{n}$;

(iii) For two samples out of three, $|\bar{z}_n|$ should be less than $1/\sqrt{n}$;

(iv) For only one sample out of 20 should it exceed $2/\sqrt{n}$;

(v) For only one sample out of 370 should it exceed $3/\sqrt{n}$.

When σ is not specified, we then have to form the ratio

$$z_s=(\bar{x}_n-\langle \mathbf{x}\rangle_{\text{Av}})/\sigma_n, \quad (42)$$

known as "Student's" z. [We have added the subscript s merely to distinguish "Student's" z from the z in Eq. (10′).] It is really a special case of "Student's" t. If the samples be drawn from a "normal" universe, Gossett[6] found:

$$\phi(z_s)=\frac{\Gamma(\frac{1}{2}n)}{\pi^{\frac{1}{2}}\Gamma[\frac{1}{2}(n-1)]}(1+z_s{}^2)^{-\frac{1}{2}n}; \quad (43)$$

$$\langle \mathbf{z}_s\rangle_{\text{Av}}=0; \quad (44)$$

$$\sigma_{z_s}{}^2=1/(n-3), \quad n\geqslant 4; \quad (45)$$

$$\alpha_{3z(s)}=0; \quad (46)$$

$$\alpha_{4z(s)}=3(n-3)/(n-5)=3+6/(n-5), \; n\geqslant 6. \quad (47)$$

Here, both the variance and the kurtosis are larger than those for the distribution of $\bar{z}_n$, in agreement with the facts that $\langle \sigma_n\rangle_{\text{Av}}$ is smaller than σ, and that the distribution of σ_n is skewed. Graphs of z_s for $n=4$ and for $n=10$ were given by Gossett.[6] The graph for $n=10$ is given in Fig. 3, and some values of $\phi(z_s)$ for $n=5$ and $n=10$ are given in Table IV.

Shewhart[11] has shown that Eq. (43) is a satisfactory approximation for samples drawn from certain non-normal universes also.

TABLE IV. "Student's" z. For 5 and 10 specimens, compared with the corresponding "normal" distributions. The corresponding "normal" distribution is the one whose standard deviation is $(n-3)^{\frac{1}{2}}$; see Eq. (45). It is given by $\psi(z)=(n-3)^{\frac{1}{2}}\psi_0[z(n-3)^{\frac{1}{2}}]$; see also Eq. (15).

	"Student's" distribution		Corresponding "normal" distribution	
z	$\phi(z_s)$	$\int_{-z}^{z}\phi(z_s)dz$	$\psi(z)$	$\int_{-z}^{z}\psi(z)dz$
	For samples of 5 specimens			
0.0	0.750	0.000	0.565	0.000
.1	.732	.148	.559	.112
.5	.430	.626	.440	.520
1.0	.133	.960	.208	.842
2.0	.013	.984	.010	.996
3.0	.002	.996	.0001	
	For samples of 10 specimens			
0.0	1.164	0.000	1.055	0.000
.1	1.109	.228	1.020	.209
.5	0.382	.830	0.440	.814
1.0	.036	.986	.032	.991
1.5	.003	.998	.0005	.99999

[11] W. A. Shewhart, *Economic control of quality of manufactured product* (Van Nostrand, 1931), p. 190.

Extremes of Sampling

Suppose that the fraction of a population having a value of x greater than some x_0 (or the fraction less than some x_0) is very small, say, 0.001. Then, suppose that one such member of the population appears in a small sample of, say, 10 or 20 specimens. The presence of this rare member unduly affects the average and the standard deviation of the sample, and, therefore, spoils the estimate of the average and standard deviation of the universe. Therefore, so it is argued by a certain school of thought, a better estimate of the properties of the universe is obtained by excluding this member from the sample.

Also, in the minds of those who argue thus, the presence of the rare member in the sample is more likely to be caused by an error in measurement than by an extreme of sampling. This, to them, is additional reason for excluding this member from the sample.

The writer has heard this line of reasoning both in industry and in the classroom, and cannot agree with it. If the experimenter believes that the estimate of universe properties obtainable from the actual data is erroneous, he should increase the sample size—obtain additional specimens—until the total sample size is such that the presence or absence of the suspected member makes but very little difference in the final result, unless he is prevented by the most cogent reasons, such as prohibitive cost, lack of time or material.

As for possible errors of measurement, data should not be rejected for statistical reasons alone; data should be rejected only when they can be impeached for valid physical reasons.

In those cases where it is necessary or permissible to reject data for statistical reasons, one must, in order to do it logically and consistently, know something about the distribution of

$$z_t = (x - \bar{x}_n)/\sigma_n,$$

where $\bar{x}_n$ and σ_n are the average and the standard deviation of the sample *before* the sample has been corrected by the exclusion of the extreme member.

In 1935, Thompson[12] showed that the distribution of z_t is closely related to that of "Student's" t for the two samples of size $(n-1)$ and 1. He did not give an explicit formula for $\phi(z_t)$, but gave the results of his work in a table.

Other treatments of this problem have been given by Tippett[10] and by Irwin.[13]

A common error among those who desire to prune their data is to assume that the distribution of z_t is normal. A well-known fallacy is based on the assumption that $\phi(z_t)$ is the same as $\psi_0(z)$.

Discrete Variables, or Attributes

So far, we have discussed the distributions of continuous variables. However, there are many universes that consist of a finite number of values or of kinds of things—for example, the numbers obtainable by rolling dice.

In the simplest cases, the universe consists of two kinds of things, P and Q, the fraction of the first kind being denoted by $\mathbf{p}$, and the fraction of the second kind, by $\mathbf{q}$. In quality control, P is the undesirable portion of the universe, and $\mathbf{p}$ is called the universe fraction defective—in any reasonable manufacturing process, it is a small fraction—and $\mathbf{q}$ is called the fraction effective. Because of its simplicity and convenience, we will use this terminology for the general case, denoting the smaller fraction of a discrete universe by $\mathbf{p}$, and calling it the *fraction defective*, whatever its nature.

Therefore, let us draw samples of n specimens each from such a universe, and let r be the number of defectives in a sample; $p_n[=r/n]$, the fraction defective of the sample; $q_n[=(n-r)/n]$, the fraction effective of the sample; and $m[=\mathbf{p}n]$, the "expected" number of defectives in the sample. Then, as is shown even in some high school algebra books, the probability that a sample of n specimens will have r defectives is

$$\phi(r) = C_r^n \mathbf{q}^{n-r}\mathbf{p}^r, \tag{48}$$

where

$$C_r^n = n(n-1)\cdots(n-r+1)/r! = n!/r!(n-r)! = C_{n-r}^n;$$

C_r^n is also sometimes written as ${}_nC_r$ and as $\binom{n}{r}$.

[12] W. R. Thompson, *Ann. Math. Statistics* 6, 214 (1935).

[13] J. O. Irwin, *Biometrika* **17**, 100, 238 (1925).

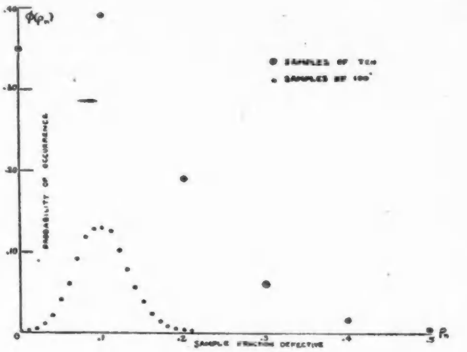

FIG. 4. Distribution of sample fractions defective, when the fraction defective of the parent universe is 0.10, for samples of 10 specimens, and for samples of 100 specimens. When $p=0.5$, the distribution of p_n is symmetrical. As n is increased, the distribution approaches "normality," even for extreme values of p.

For example, suppose that $\mathbf{p}=0.1$, and that we draw many samples of 10 specimens each from the universe. Then, 35 percent of the samples will have no defectives, 39 percent will have one defective, and 26 percent will have two defectives or more. Thus, 74 percent of the samples will be as good as the universe or better, whereas only 65 percent will be as bad as the universe or worse. But, if we take samples of 100 specimens each, the disparity will not be so bad, for 49 percent of these samples will have fewer than 10 defectives, 13 percent will have exactly 10 defectives, and 41 percent will have more than 10 defectives.

Thus, small samples from a reasonably good lot generally make the lot appear better than it really is; the better the lot and the smaller the samples, the worse the deception (in magnitude, not necessarily in seriousness). See Fig. 4.

The sample fractions defective, p_n, constitute a subsidiary universe of $(n+1)$ discrete values (from 0 to unity), as do the sample fractions effective and the number of defects in the samples (from 0 to n). The averages and variances of these subsidiary distributions are:

$$\langle \mathbf{p}_n \rangle_{Av} = \mathbf{p}, \quad \langle \mathbf{q}_n \rangle_{Av} = \mathbf{q}, \quad \langle \mathbf{r} \rangle_{Av} = m; \tag{49}$$

$$\sigma_{p_n}{}^2 = \mathbf{pq}/n = \sigma_{q_n}{}^2, \quad \sigma_r{}^2 = m(n-m)/n, \tag{50}$$

where

$\sigma_{p_n}{}^2$ is the average value of $(p_n-\mathbf{p})^2$,
$\sigma_{q_n}{}^2$ is the average value of $(q_n-\mathbf{q})^2$,
$\sigma_r{}^2$ is the average value of $(r-m)^2$.

In most practical problems (but in the next section is an important exception), the relative values of $\mathbf{p}$, $\mathbf{q}$, m, n and r are such that

$$C_r{}^n \mathbf{q}^{n-r} \mathbf{p}^r \rightarrow m^r e^{-m}/r! \tag{48'}$$

and

$$\sigma_{p_n}{}^2 = \sigma_{q_n}{}^2 \rightarrow \mathbf{p}/n = m/n^2, \qquad \sigma_r{}^2 \rightarrow m. \tag{50'}$$

Equation (48′) is known as Poisson's exponential approximation. It is surprisingly close. It is independent of the sample size n. According to Poisson's formula, the probability of obtaining a "perfect" sample ($r=0$), when the "expected" number of defectives is m, is e^{-m}—an expression easy to remember, and helpful in many ways when dealing with attributes.

Sampling or testing by attributes should be employed only if there is no continuous variable to measure. Unfortunately, there has become established in many manufacturing industries the practice of controlling even simple dimensions, such as lengths and diameters, by "go-not-go" gages. This practice is unsound and inefficient. It is both more informative and more economical to measure small samples at suitable intervals than to gage large samples, for controlling any process involving continuous variables.

Extensive tables for interpreting the results of drawing samples from universes of discrete variables have been prepared by Dodge and Romig.[14]

Cumulative Distributions

Sometimes it is not possible to determine what portion of a sample lies between x and $x+\Delta x$. Measurements of sensitivity of explosive mixtures and of dosage effect among guinea pigs are examples. One can apply a given stimulus or impulse to a sample of n specimens of an explosive mixture, and record the fraction of the sample that responds in some desired way, and the fraction that fails to so respond. One can administer a given dose to a sample of n guinea pigs, and record the fraction of the sample that is affected in some manner, and the fraction that is not. But one can never determine the exact impulse required to fire a given ammunition primer, nor the exact dose required to affect a given guinea pig. The experiment cannot, as a rule, be repeated on the surviving members, as they are not left in the initial condition after the initial test.

[14] H. F. Dodge and H. G. Romig, *Sampling inspection tables* (Wiley, 1944).

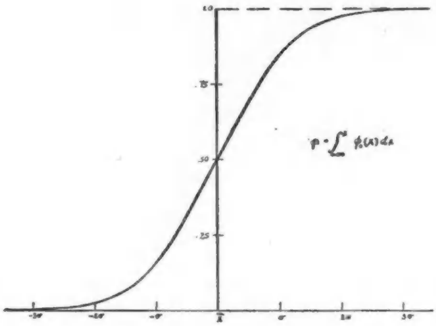

FIG. 5. Graph of the cumulative frequency distribution,

$$p=\int_{-\infty}^{x}\phi(x)\mathrm{d}x.$$

Such a graph is called the ogive of the parent distribution. For the normal distribution, the tangent to the ogive at $\langle \mathbf{x}\rangle_{Av}$ is equal to $1/\boldsymbol{\sigma}(2\pi)^{\frac{1}{2}}$.

However, additional samples can be selected, and the test repeated with a different impulse or a different dose. Then, let the reciprocal of the sensitivity (let us call it the insensitivity, for convenience) of either the primers being tested, or of the guinea pigs being used to determine the strength or potency of a drug, be denoted by x, where x is measured by the magnitude of the impulse or dose required to produce the desired effect; and let the distribution of x among the primers or guinea pigs be denoted by $\phi(x)$. Then, the fraction of the universe of primers or of guinea pigs that will respond in the desired manner to an impulse or dose of magnitude x is

$$\mathbf{q}=\int_{x}^{\infty}\phi(x)\mathrm{d}x,$$

and the fraction failing to respond is

$$\mathbf{p}=\int_{-\infty}^{x}\phi(x)\mathrm{d}x. \qquad (51)$$

A graph of Eq. (51) for the case in which $\phi(x)$ is the "normal" distribution function is given in Fig. 5.

The observed values of q_n and p_n for each sample tested are the "best estimates" of $\mathbf{q}$ and $\mathbf{p}$ for each value of x.

A method for determining the moments of $\phi(x)$ from the observed q_n and p_n has been developed by Epstein and Churchman.[15] If we test k samples of n specimens each (the n's need not be all alike) at equally spaced intervals of x, and if $p_{n_1}=1$, $p_{n_k}=0$, the equations for the average and the variance of the insensitivity of these specimens reduce to

$$\bar{x}_{k,n}=x_1+\tfrac{1}{2}\Delta x+\sum_{2}^{k-1} p_n\Delta x, \qquad (52)$$

$$\sigma_{k,n}^2=\left[\sum_{2}^{k-1}\beta_2 p_n-\left(\sum_{2}^{k-1}p_n\right)^2\right](\Delta x)^2, \qquad (53)$$

where the β_2's are the differences between the successive squares (in other words, the successive odd integers). The equation for the third moment reduces to

$$\mu 3_{k,n}=\left[\sum_{2}^{k-1}\beta_3 p_n-3\left(\sum_{2}^{k-1}\beta_2 p_n\right)\left(\sum_{2}^{k-1}p_n\right)+2\left(\sum_{2}^{k-1}p_n\right)^3\right](\Delta x)^3, \qquad (54)$$

where the β_3's are the differences between the successive cubes (1, 7, 19, 37, 61 $\cdots$). The expressions for the higher moments are similar. (See Example 2, Appendix A.)

Now, each time we conduct the test, we expect to get somewhat different values for $\bar{x}_{k,n}$ and for $\sigma_{k,n}^2$ and for the higher moments. Thus, the sample moments form subsidiary universes, as before.

The variance of the universe of averages is the sum of the variances of the universes of the k samples tested. From Eq. (50) we obtain

$$\boldsymbol{\sigma}_{\bar{x}_{k,n}}^2=\sum\boldsymbol{\sigma}_{p_n}^2(\Delta x)^2=\left(\sum\mathbf{pq}/n\right)(\Delta x)^2,$$

and the "best estimate" obtainable from these k samples is

$$\boldsymbol{\sigma}_{\bar{x}_{k,n}}^2=\sum_{2}^{k-1}\sigma_{p_n}^2(\Delta x)^2=\left(\sum_{2}^{k-1}p_n q_n/n\right)(\Delta x)^2. \qquad (55)$$

Similarly, the "best estimate" of the variance of the universe of sample variances is

$$\boldsymbol{\sigma}_{\sigma(k,n)^2}^2=\left[\sum_{2}^{k-1}\left(\beta_2-2\sum_{2}^{k-1}p_n\right)^2\sigma_{p_n}^2\right](\Delta x)^4. \qquad (56)$$

Equations (55) and (56) are required for determining whether the variations from test to test are consistent with the variations within the tests.

[15] B. Epstein and C. W. Churchman, *Ann. Math. Statistics* **15**, 90 (1944).

Because of the importance, difficulties and expense of tests of this kind, adequate and thorough statistical analysis is perhaps more important than in more conventional tests (in which the fraction lying between x and $x+\Delta x$ can be determined directly). Particularly in tests of dosage mortality and of explosive sensitivity is it important to be able to tell whether the difference between two tests connotes a real change in the product (drug or explosive) or a mere sampling fluctuation.

Appendix A

Example 1. Calculation of the first four moments.—The moments of the sample are defined as follows:

zeroth moment, or *average*, $\bar{x}_n=\sum_n x/n$;

second moment, or *variance*,

$$\sigma_n^2=\sum_n (x-\bar{x}_n)^2/n=\sum_n x^2/n-(\bar{x}_n)^2;$$

third moment,

$$\mu 3_n=\sum_n (x-\bar{x}_n)^3/n=\sum_n x^3/n-3\bar{x}_n\sum_n x^2/n+2(\bar{x}_n)^3;$$

fourth moment,

$$\mu 4_n=\sum_n (x-\bar{x}_n)^4/n=\sum_n x^4/n-4\bar{x}_n\sum_n x^3/n +6(\bar{x}_n)^2\sum_n x^2/n-3(\bar{x}_n)^4.$$

Each specimen is carefully measured, and the number of specimens having measurements lying within convenient intervals Δx is determined; Δx may be of any convenient size from the limit of measurement up. In the example given, the limit of measurement was 0.0001 in. (with micrometer caliper), but the number of specimens in such small intervals was small, and the number of intervals was too large. The measurements were therefore grouped into intervals, or "cells," 0.0005 in. wide. For the purposes of computation, all the specimens within a cell are considered to have the measurement of the center of the cell. This is comparable to measuring the specimens to the nearest 0.0005 in. in the first place; but one does not always know beforehand what the convenient interval will be, so that it is prudent to measure as closely as possible, and to regroup the measurements, if desirable, afterwards.

Let the number of specimens in each cell be denoted by N, so that $\sum_k N=n$. Then

$$\sum_n x/n\equiv\sum_k Nx/n,\quad \sum_n x^3/n\equiv\sum_k Nx^3/n,$$
$$\sum_n x^2/n\equiv\sum_k Nx^2/n,\quad \sum_n x^4/n\equiv\sum_k Nx^4/n,$$

where x is the mid-point of each cell, and k is the number of cells.

Next, to further simplify the calculation, choose the center of the cell nearest to the apparent center of the sample as the origin of new coordinates, and let the distance between the center of each cell and this origin be denoted by X; X is therefore an integral multiple of Δx. If the center of the cell taken as origin be denoted on the original coordinates by x_0, then

TABLE IA.

Interval or cell	X	N	NX	NX^2	NX^3	NX^4
0.4996–0.5000	−5	1	−5	25	−125	625
.5001– .5005	−4	5	−20	80	−320	1280
.5006– .5010	−3	39	−117	351	−1053	3159
.5011– .5015	−2	60	−120	240	−480	960
.5016– .5020	−1	86	−86	86	−86	86
.5021– .5025	0	85	0	0	0	0
.5026– .5030	1	93	93	93	93	93
.5031– .5035	2	77	154	308	616	1232
.5036– .5040	3	38	114	342	1026	3078
.5041– .5045	4	13	52	208	832	3328
.5046– .5050	5	1	5	25	125	625
.5051– .5055	6	2	12	72	432	2592
Totals		500	82	1830	1060	17058

$$x=x_0+X,$$
$$\bar{x}_n=x_0+\bar{X}_n=x_0+\sum_k NX/n,$$
$$\sigma_n^2=\sum_k N(x-\bar{x}_n)^2/n=\sum_k N(X-\bar{X}_n)^2/n =\sum_k NX^2/n-(\bar{X}_n)^2,$$

and so on for the higher moments. Of course, in these equations, x, x_0 and $\bar{x}_n$ must be expressed in the same units as X.

The example shown is from the control of a high speed mass-production process. A machine is producing 200 pieces each hour. It is desired that one of the dimensions of these pieces, when finished, be between 0.5000 and 0.5050 in. It is also desired to control the manufacturing process, once it is established, by measuring small samples (5 or 10) two or three times each hour. The objects of the test shown in the example were: (1) to determine whether it is possible for the machine being used to keep the dimension within the desired limits; (2) to determine the most economical sampling scheme if the machine be found satisfactory. After the machine had been running for a while, so that a sort of "steady state" had been reached, a sample of 500 specimens was delivered (Table IA). For this sample:

$$x_0=0.5023 \text{ in.};$$
$$\bar{X}_n=\sum NX/n=82/500=0.164\times 0.0005 \text{ in.}=0.000082 \text{ in.};$$
$$\bar{x}_n=x_0+\bar{X}_n=0.5023+0.000082=0.5024 \text{ in.};$$
$$\sigma_n^2=\sum NX^2/n-(\bar{X}_n)^2=1830/500-(0.164)^2 =3.66-0.027=3.63\times(0.0005 \text{ in.})^2;$$
$$\sigma_n=\sqrt{3.63}=1.906\times 0.0005 \text{ in.}=0.00095 \text{ in.}=0.0010 \text{ in.};$$
$$\mu 3_n=\sum NX^3/n-3\bar{X}_n\sum NX^2/n+2(\bar{X}_n)^3=1060/500 -3(0.164)(3.66)+2(0.164)^3=0.328\times(0.005 \text{ in.})^3;$$
$$\alpha 3_n=\mu 3_n/\sigma_n^3=0.328/(1.906)^3=0.047, \text{ a pure number};$$
$$\mu 4_n=\sum NX^4/n-4\bar{X}_n\sum NX^3/n+6(\bar{X}_n)^2\sum NX^2/n-3(\bar{X}_n)^4 =17058/500-4(0.164)(2.30)+6(0.164)^2(3.66) -3(0.164)^4=33.31\times(0.0005 \text{ in.})^4;$$
$$\alpha 4_n=\mu 4_n/\sigma_n^4=33.31/(3.63)^2=2.52, \text{ a pure number}.$$

From Eq. (25), we see that $\sigma_{\alpha 3(n)}$ is $\sqrt{(6/n)}=\sqrt{(6/500)}=0.11$. The skewness of the given sample is less than half this. As a rough approximation, therefore, only two-fifths

TABLE IIA.

x	Failures	p_n	β_2	$\beta_2 p_n$	β_3	$\beta_3 p_n$
5	50	1.00	0	0.00	0	0.00
6	48	0.96	1	.96	1	.96
7	33	.66	3	1.98	7	4.62
8	14	.28	5	1.40	19	5.32
9	4	.08	7	0.56	37	2.96
10	1	.02	9	.18	61	1.22
11	0	.00	11	.00	91	0.00
Totals		2.00		5.08		15.08

of all the samples of 500 specimens drawn from a normal universe will have $\alpha 3_n$ lying between 0.047 and -0.047. The figure for most other symmetrical universes will not be widely different. Therefore, in this case, the parent universe is probably symmetrical.

Fewer than 1 percent of all the samples of 500 drawn from a normal universe will have kurtoses $\alpha 4_n$ as small as 2.52. Thus, the distribution curve for the parent universe, in this case, is probably flatter than normal. This is frequently a symptom of some systematic, progressive change (such as tool wear) during the process. There are numerous ways of verifying this assumption; thus, if direct measurements of the tools are not possible, the average of the first 50 specimens of the sample might be compared with that of the last 50 produced. Such a comparison would indicate merely the probable presence or absence of such a trend.

Since σ_n is 0.0010 in., and the width of the tolerance band is 0.0050 in., or $5\sigma_n$, then approximately 1 percent of the production will be outside the desired limits. The sample of 500 had only 3 specimens outside the desired limits, but the inference drawn from the sample as a whole—from the moments—is more reliable than that drawn from the tails of the sample. One would therefore conclude that the present process is not good for the job.

However, if the supposition about the systematic trend be correct, σ can be reduced by readjusting the tools (or other cause of the systematic trend) at suitable intervals—let us say, at the end of each hour. This amounts to making the parent universe the potential product of an hour's run, instead of that of an indefinite period.

Example 2. Sensitivity data.—For each value of x given in the first column of Table IIA, 50 specimens were tested. The number that failed to respond in the desired manner is given in the second column, and the fraction p_n failing to respond is given in the third. In the fourth column are given the β_2's, and in the fifth, the product of each β_2 and its p_n. These five columns suffice for the calculation of $\bar{x}_{k,n}$ and $\sigma_{k,n}$. This is usually all that need be calculated.

When it is desired to calculate higher moments, it is merely necessary to add two columns for each higher moment desired: one for the corresponding β, and one for the products βp.

$$\Delta x = 1,$$

$$\bar{x}_{k,n} = x_1 + \Delta x/2 + \Sigma p_n \Delta x = 5 + \tfrac{1}{2} + 2.00 = 7.50;$$

$$\sigma_{k,n}^2 = [\Sigma\beta_2 p_n - (\Sigma p_n)^2](\Delta x)^2 = 5.08 - (2.00)^2 = 1.08;$$

$\sigma_{k,n} = \sqrt{1.08} = 1.04$, in the same units as x;

$$\mu 3_{k,n} = [\Sigma\beta_3 p_n - 3(\Sigma\beta_2 p_n)(\Sigma p_n) + 2(\Sigma p_n)^3](\Delta x)^3$$
$$= 15.08 - 3(5.08)(2.00) + 2(2.00)^3 = 0.60;$$

$$\alpha 3_{k,n} = \mu 3_{k,n}/\sigma_{k,n}^3 = 0.60/1.12 = 0.54.$$

Appendix B: Bibliography

This list is far from exhaustive: it includes merely the books with which the writer happens to be familiar and that he has used at some time or other. Every experimenter should have one or more of these in his library. Fuller bibliographies may be found in many of them.

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[*A second article, on practical methods of applying the principles outlined, will appear in the next issue.*]

The Perception of Objects

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For more than a century it has been customary to say that perception is something more than sensory impression, that perception is *of an object*, that it corresponds to a stimulating object. The modern view is that, because objects are permanent, a perception of an object tends to remain constant even when the immediate sensory impressions upon which the perception is based vary with the variety of conditions that affect stimulation.

This general rule of perception applies to all sense departments. It depends upon an integrative property of the brain and is not a function of sense organs at all. The meaning of the rule is most easily expounded in terms of particular instances, and the four examples that are best understood are the visual perception of size with distance variant, of shape with angle of regard variant, of brightness with intensity of illumination variant, and of hue with color of illumination variant.[1]

We know a great deal about perceived *size* with distance variant. At short distances perceived size tends not to change with distance. A man is 40 ft down the hall and walking toward you. When he is only 20 ft away, has he doubled his height? The height of the image of him on your retinas has doubled. The perception itself, however, changes very little. Or you are at a reception, standing at the end of a large hall. Are the people at the far end dwarfs, only half as tall as the people in the middle of the hall, only a tenth or a twentieth as tall as the man with whom you are talking? Do people change in size at the rate at which the images of them on your retinas change?

What happens is that, under certain circumstances, the brain corrects the perception that depends initially upon the size of the retinal image, corrects it in accordance with other sensory data that indicate the distance from which the retinal image is projected. And the brain can do an excellent job in this kind of correction. At great distances, however, the corrective mechanism becomes inadequate. A man a mile away actually does look small, in part because a mile cannot be perceived accurately. On the other hand, there is doubtless some cerebral correction for the size of a man a mile away, for even the moon—239,000 mi away—shows a tiny correction for the smallness of its retinal image caused by its great distance.

This same tendency to preserve objective constancy happens with visually perceived *shape*. As I stand to one side and look at the top of a circular table, it does not appear as the narrow ellipse that its retinal image is, that the artist would sketch in his projection of the scene. Although every room is full of rectangles, they are perceived not as various diamonds and distorted rectangles, but approximately in their true proportions. The brain corrects the perception for the angle of projection.

So it goes with *brightnesses*. Coal looks black and white paper looks white, provided you know that you are seeing coal and white paper. The white paper may be in shadow and the coal in bright illumination, illumination so bright that the coal reflects more light than the paper. Still the coal looks black and the paper white. The brain takes account of the nature of the objects and corrects the initial impressions that are based solely on illumination.

Sir Isaac Newton observed this phenomenon. He was put to it to prove that gray is a darkish white, because gray things persist in looking gray and white things white. So he rubbed a gray powder on the floor of his chamber in the sunlight and laid nearby in the shadow a piece of white paper. Then he viewed the two objects from a distance so great that their character as objects could not be recognized and saw that the gray powder in sunlight was as white as or whiter than the white paper in shadow. A friend, pressed into

[1] On constancy in the perception of objects, see, for elementary accounts, E. G. Boring, H. S. Langfeld and H. P. Weld, *Introduction to psychology* (Wiley, 1939), pp. 420–427, 463f., 468f.; R. S. Woodworth, *Experimental psychology* (Holt, 1938), pp. 595–622. Another summary of the facts with historical orientation and two score references is in E. G. Boring, *Sensation and perception in the history of experimental psychology* (Appleton-Century, 1942), pp. 254–256, 262, 288–299, 308–311. The fullest and most technical discussion of the literature is in K. Koffka, *Principles of Gestalt psychology* (Harcourt Brace, 1935), pp. 211–264.

service as an observer and asked to judge the patches of brightness before he knew what the objects were, corroborated him.[2]

The general rule also holds for ***hue***. If you have the means of knowing from other immediately present sensory data or from past experience what color an object ought to be, then you are apt to see it in its correct color whatever the hue of the illumination. Familiar dresses and upholstery may keep their daylight hues in yellowish artificial illumination, but a new observer with no familiarity to guide him will see them with the yellows favored and the blues diminished. Twenty years ago the technicolor motion pictures were using only two component hues to make colors that should have been trichromatic. One color was put on each side of the film, and the film had only two sides. It was the blues that were cheated. The colors used were a slightly bluish red and a slightly bluish green, which will mix to give good reds and greens, poor yellows and very poor blues. What did the audiences, unused in those days to colored movies, say? That the American flag was beautiful, that the (bluish-green) skies were lovely. But the heroine never wore a pure blue dress (whatever she had on in the studio) because dresses, unlike the sky or the flag's field, can be any color and obey the laws of color mixture without this kind of cerebral mediation. Yet Little Boy [Greenish] Blue in the old-fashioned technicolor might have looked as blue as the sky.

Reduction and Regression

For descriptive purposes it is convenient to say that the sensory data that contribute to a perception can be divided into a core and its context. The *core* is the basic sensory excitation that identifies the perception, that connects it most directly with the object of which it is a perception. The *context* consists of all the other sensory data that modify or correct the data of the core as it forms the perception. The context also includes certain acquired properties of the brain, properties that are specific to the particular perception and contribute to the modification of its core. In other words, the context includes knowledge about the perceived object as determined by past experience, that is, by all the brain habits which affect perceiving.[3]

In visual perception the core is the retinal excitation, that is to say, the total optical pattern, specified with respect to the wavelengths and energies involved and the spatial distribution and temporal changes of each. Thus in the visual perception of size with distance variant, the core is the size of the retinal image. The context includes all the clues to the distance of the perceived object—clues of binocular parallax and convergence, and of lenticular accommodation and perspective, as well as the other monocular clues to the awareness of distance. If an observer has before him one disk 10 ft away and another 20 ft away, and undertakes to alter the size of one until it looks the same size as the other, he is likely to come out with two disks of the same physical size. He is obviously not then equating retinal images, for the image of the farther disk has a diameter only half that of the image of the nearer. His brain is using his awareness of distance to make the perception derived from the smaller retinal image look as large as the perception from the larger retinal image. (Whether this correction is an inference, a physiological process or both is a matter that we must consider presently.)

If the perceived object is not a disk, which is unprejudiced as to size, but a man, whose height is, of course, likely to lie between 5 and 7 ft, then this special knowledge is added to the context. Such contextual knowledge does not, however, necessarily prevail. If the visual afterimage of a 6-ft man who is 20 ft away is projected on a wall 60 ft away, the man in the afterimage will be a giant, not far from 18 ft tall—provided the observer is able to perceive the distance to the wall.

These facts of perception can be demonstrated by the *reduction* of context. When distance is fully apprehended, the perceived size of an object is likely to remain about constant, even though the distance changes greatly and the size of the retinal image changes with it. On the other hand, if you reduce the clues from which distance can be gaged, then you will find that perceived size

[2] I. Newton, *Opticks* (1704; reprint, Bell, 1931), bk. I, pt. ii, prop. v, exp. 15.

[3] This convenient distinction between core and context derives from the one first made by E. B. Titchener, *Textbook of psychology* (Macmillan, 1910), pp. 367–371; *A beginner's psychology* (Macmillan, 1915), pp. 114–121.

changes with distance, getting smaller at greater distances. When this reduction of context is partial, the shrinkage of the perception with increasing distance will be less than the shrinkage of the corresponding retinal image. If the reduction could be made complete, if all clues to distance other than the changing size of the retinal image were eliminated, then presumably perceived size would be determined by the only clue remaining, the size of the retinal image. Perceived size and retinal size would then always keep the same proportional relation. With a receding object perceived size would shrink as fast as the retinal image, for the observer would be wholly unaware of the recession, unless he used his awareness of diminishing size as a clue to the perception of the distance. He might.

Some writers have preferred to think of the converse of this relationship. Reduction of context reduces perception to its bare core, but increase of context increases correspondence of the perception to the real object. If you know how far away a seen object is, you may be able to perceive its true size. If you know that that gray is coal, it may look black. This effect of context to make the perception resemble the permanent object that is being perceived rather than the perceptual core has been called by Thouless *regression toward the real object*.[4] Regression toward the object is the opposite of reduction toward the core. Regression toward the object occurs with increase of context. Reduction toward the core is secured by decrease of context.

It was Katz who in 1911 first applied this term *reduction*, not to the case of size and distance, but to color and illumination.[5] Take the illuminated coal and the shaded white paper. The coal looks black, the paper white, although the coal reflects more light. But now interpose what Katz called a *reduction screen*, a screen with two small circular openings in it. Through one opening you see a patch of the surface of the coal, through the other a patch of the surface of the paper. You do not see enough to identify either object, and at once the patch that is the coal appears as a gray lighter than the patch that is the white paper. By the use of the screen you have reduced the context that identifies the objects, reduced it to the core of this perception, the total illumination.

Perceived Size and Distance: History

Most persons think that the perceived size of an object varies proportionately with retinal size of its image and thus with the visual angle subtended by the object. There seem to be two reasons for this belief.

(1) To assume that visual size is measured by visual angle brings perception into the geometry of optics in a simple and logical way. Euclid in his *Optica* worked in terms of this optical geometry, equating visual angle to perceived size; yet even he noted that the magnitude of the perception does not always accord with the perceived size. Still Euclid provided the simple rule that has ever since been quite generally accepted. The principle that perceived size varies with the visual angle subtended and thus with the size of the retinal image we shall call *Euclid's rule*.

(2) Progress in understanding visual perception during the last three centuries has consisted primarily in finding out how the eye works. From the ancient belief that objects give off tiny images of themselves, images which are conducted by the optic nerve to the sensorium for perception, we have come to an understanding of optical projection upon the retina. In general, nineteenth-century physiology held that the brain perceives not the object itself but its projection on the retina and the consequent excitation of the optic nerve. (Why else should there have been a problem as to why we see right side up when the retinal image is upside down?) When perception was found not to correspond exactly with the stimulus object, one looked—all through the nineteenth century—to the eye for the explanation of the illusion, for but little was known about the supraretinal physiology of vision.

Nevertheless, there have been constant reminders that visual angle (retinal size) and perceived size do not always correspond. There were the eighteenth-century philosophers who tried to figure out the curve along which trees, bordering on an avenue, should be planted in order that the two lines of trees might look parallel and everywhere equidistant, when viewed from a specified

[4] R. H. Thouless, "Phenomenal regression to the real object," *Brit. J. Psychol.* **21**, 339–359 (1931); **22**, 1–30 (1931).

[5] D. Katz, "Die Erscheinungsweisen der Farben und ihre Beeinflussung durch die individuelle Erfahrung," *Z. Psychol.*, Suppl. **7**, esp. 36–39 (1911).

end. And there were the psychologists in the period 1889–1913 who determined these curves experimentally. Various scientists remarked in the middle nineteenth century that Euclid's rule does not hold. If you view a length l at a distance d and compare it with a length $2l$ at a distance $2d$, then l and $2l$ form retinal images of the same size and should, by Euclid's rule, appear to be the same size. Actually, it is easily noted that $2l$ looks longer than l; yet that observation seldom excites curiosity since, it can be said, if $2l$ really is longer than l, why should it not look longer? Euclid gave a reason why it should not, but plainly he was wrong.

Then there was Emmert's law in 1881, the law that the perceived linear size of an afterimage is proportional to the distance of the background on which the image is projected. Emmert's law contradicts Euclid's. In an afterimage the size of the retinal image is fixed; how then does the size of the perception alter so greatly? Yet the fact is that the afterimage of a near object projected on a far background looks gigantic when retinal size does not change at all.

In the present century there has arisen in connection with the tenets of Gestalt psychology the conception of *size constancy*, the hypothesis that the perceived size of an object remains constant irrespective of the distance at which the object is perceived. There has been a great deal of misunderstanding and controversy about this matter; but that need not concern us, for the facts are plain.[6]

Perceived Size and Distance: Measurement

Let me now summarize an experiment that Holway and I completed a few years ago, an experiment that measures the dependence of perceived size upon distance, and that also shows how the resulting functions depend upon the context and how the effective context can be analyzed by successively reducing it.[7]

We seated the observer at the right-angled junction of two halls, facing the convex corner at 45° from the longitudinal axes of the halls. By turning his head from side to side he could look down one hall or the other, alternating at will. We worked at night in complete darkness except for the light that came from the two illuminated stimulus disks.

The standard stimulus was a disk of light, projected from a lantern on a translucent screen. It was placed in the hall at the right at distances from the observer varying from 10 to 120 ft. The size of the disk was made proportional to the distance so that it always subtended at the observer an angle of 1°. It was the perceived size of this disk at different distances that was the subject of investigation.

In the hall at the left was a comparison stimulus for measuring perceived size. It consisted of a projected disk of light which remained always at 10 ft from the observer. Its size could be changed by the use of a long series of apertures with which the projection lantern was provided. The observer varied the size of this comparison stimulus until he was satisfied that he had made it the same perceived size as the standard stimulus. This judgment is not always easy when the disks are at different distances, but the difficulty arises only in the final adjustments of the comparison stimulus. Great differences in perceived size at great differences in distance are easily observed with immediate certainty, even when the judgment departs widely from Euclid's rule of the visual angle.

The results of this procedure are plotted in Fig. 1. The two dashed lines, B and F, represent theoretical relationships.

The function B is for perceived size constant, irrespective of distance. The reason this line rises (slope = tan 1°) is that we increased the size of the standard stimulus in proportion to its distance, keeping its angular subtension constant at 1°. We did this in order to avoid any physiological distortion that might arise from exciting different sizes of retinal areas. The straight line B is thus the function for perceived *size constancy* because it is the function for a comparison stimulus increasing with the standard stimulus and remaining equal to it, when the standard stimulus is increased with distance in order to keep angular subtension constant.

The horizontal line F is the function for proportionality of perceived size to retinal size (visual angle). It represents *Euclid's rule.* Since

[6] For the history of research and observation, see Boring, reference 1 (1942), pp. 288–299, 308–311.

[7] A. H. Holway and E. G. Boring, "Determinants of apparent visual size with distance variant," *Am. J. Psychol.* **54**, 21–37 (1941).

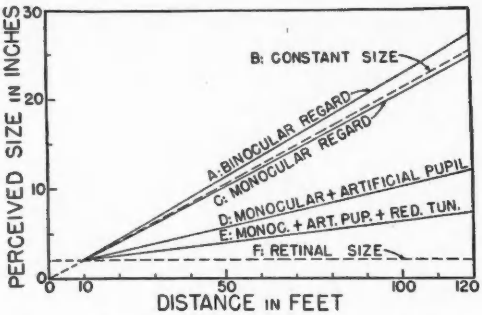

FIG. 1. Visually perceived size as a function of the distance of the perceived object. The perceived standard disk at every distance is of such physical size that its diameter subtends an angle of 1° to the observer. Its perceived size (ordinate scale) is the diameter, in inches, of a comparison disk 10 ft from the observer and equated in perceived size to the standard disk. The six functions shown are as follows: *A*, free binocular regard; the perceived size increased slightly with distance; *B*, constant size, the function for no change of perceived size with distance; *C*, monocular regard; the perceived size diminished slightly with distance; *D*, monocular regard with use of artificial pupil; great decrease of perceived size with distance; *E*, monocular regard, with use of artificial pupil and reduction tunnel; still greater decrease in perceived size; *F*, retinal size, the function for decrease in perceived size proportional to the actual size of the retinal image (visual angle subtended).

the visual angle was kept constant at 1°, proportionality of perceived size to visual angle would in this case be constant.

The other four functions show what happens with reduction. Although the observed points are not shown here, the fits to straight lines are close.

Free binocular regard is shown in function *A*. If size constancy were to be the rule, we should expect *A* to coincide with *B*. To our surprise it lay slightly above *B*, for the four more reliable of the five observers. A receding object tends (under these conditions) to get a little larger in perception while its retinal image is becoming very much smaller. Perceived size, in other words, is not only constant; it is "more than constant!" The position of this function suggests that the binocular mechanism is set to compensate immediately for shrinkage of the retinal image by increasing distance and that under these conditions it overcompensates slightly. That finding might be an argument for the phenomenon's being the consequence of a property of the brain and not a matter of inference. It is doubtful, however, that any good can come from trying to distinguish between inference and its brain physiology.

Our first step was to reduce binocular to monocular vision by putting a patch over one of the observer's eyes. Function *C* resulted, a close approximation to size constancy. Some of the individual functions for *C* lay just above *B*, some below, and the average, as shown, was a little below. Later experiments by Taylor and myself with two men, each of whom had lost the use of one eye more than ten years earlier, gave functions similar to those on which *C* in Fig. 1 is based.[8] For this reason we assumed that monocular vision, either temporary or permanent, follows closely the law of size constancy, and that overcompensation may sometimes result when the use of a second eye is added.

The next reduction of the distance context was to add to monocular observation an artificial pupil, which eliminated accommodation of the iris diaphragm and reduced the effectiveness of the lenticular accommodation by stopping the lens down to 1.8 mm. With this situation there were more individual differences among observers, but the reliability of each observer remained high. Function *D* in Fig. 1 is the average result. It shows that reduction toward the proportionality to retinal size (Euclid's rule, function *F*) has progressed greatly.

The use of monocular vision with an artificial pupil still left some visible clues to the distance of the stimulus. The faint light from the two stimuli showed vaguely to the dark-adapted eye the perspective of the walls, floor and ceiling of the corridor. To get rid of these clues we constructed a "reduction tunnel" of heavy black cloth, 3 ft square in cross section and capable of being extended to 100 ft long. The observer viewed the standard stimulus through it, and a further reduction resulted, as shown by function *E* in Fig. 1.

Complete reduction should give an observed function that coincides with *F*. We were unable to obtain it. Not with all this effort and artifice could we make Euclid's commonly accepted law of perceived size as proportional to visual angle come true! Yet the function *F* must be found if all clues to distance are eliminated. The results plotted in *E* show the observers discriminating

[8] D. W. Taylor and E. G. Boring, "Apparent size as a function of distance for monocular observers," *Am. J. Psychol.* **55**, 102–105 (1942).

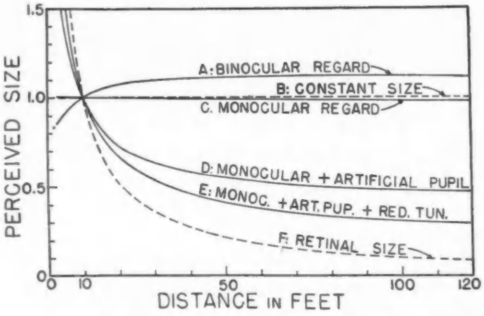

FIG. 2. Visually perceived size as a function of the distance of the perceived object. These curves are derived from those of Fig. 1 by the use of certain simple assumptions. Perceived size is shown as ratios to the perceived size of the stimulus object at a distance of 10 ft and subtending an angle of 1°. See legend to Fig. 1 for further specification of the six functions shown.

different distances fairly accurately. If the function E were known, then the distance could be told from a knowledge of the equations of perceived size. The failure to reduce to F means that there are still clues to distance left.

Figure 1 is confusing because the standard stimulus was kept constant in angular size instead of in linear size. Figure 2 is simpler, showing how the size of a constant disk would appear to change if it approached the observer or receded from him. The contractions and expansions are shown as ratios to the linear size of the disk at 10 ft away from the observer. Figure 2 is derived from Fig. 1 by making the assumption that the relationships of Fig. 1 would hold for areas of the retina differing considerably from 1°, and the further assumption that difference in perceived size is proportional to the amount of change necessary in the stimulus to abolish difference in perceived size. This last statement means, for example, that if two disks at different distances looked the same size and one had a diameter four times the diameter of the other, then, if the diameters were made actually equal, one of them would appear to be one-quarter the length of the other. It is a plausible assumption, but not necessarily correct. The functions of Fig. 2 yield curves because they are reciprocal to linear functions.

In view of these consistent results there cannot be much doubt that perceived size depends upon more than the size of the retinal image, that the clue from the retinal image is corrected by those contexts that establish the distance of the perceived object, and that the correction can be fully adequate when the context is not too greatly reduced. Monocular vision in the near-dark is good enough to keep size constant. Binocular vision in the near-dark may do too good a job. We have no measurements to show what happens with the perspective clues that good illumination in a furnished hall would produce.

These functions hold up to a distance of 120 ft, and possibly up to 200 ft. What happens at great distances? We have only casual observation and one experiment to show. A man a mile away looks small, even though seen with two eyes in broad daylight over a terrain that furnishes excellent perspective clues. Is he reduced as much in perceived size as is his image on my retina? We do not know. The experiments on great distances have yet to be made, and should be made.

There is, however, one experiment on the perceived size of a distant object—the moon. The moon's disk subtends about $\frac{1}{2}$°, but the horizon moon may be matched to a comparison stimulus that is 12 ft away and that subtends an angle of 3°, and a moon in elevation to a comparison stimulus of 2°.[9] In other words, a disk, 2160 mi in diameter and 239,000 mi away, appears the same

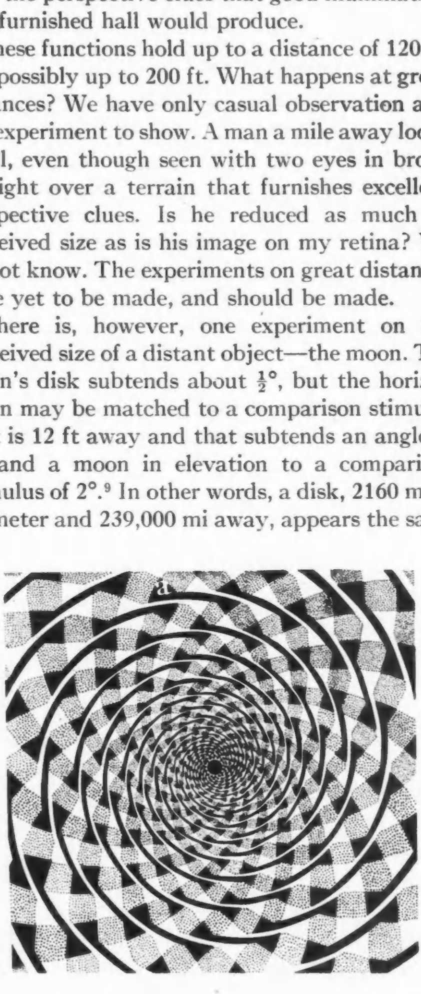

FIG. 3. Twisted cord illusion. The perceived spirals are actually perfect circles, as can be found by starting at a and following the apparent spiral back to a again. [Adapted from J. Fraser, *Brit. J. Psychol.* 2, 307–320 (1908).]

[9] E. G. Boring, "The moon illusion," *Am. J. Physics* 11 55–60 (1943), and references there cited.

size as a disk $7\frac{1}{2}$ in. in diameter 12 ft away. If Euclid's law held, the moon would match in size a disk $1\frac{1}{4}$ in. in diameter at 12 ft distance, because this little disk and the moon would each subtend an angle of $\frac{1}{2}°$. There is for the moon no size constancy. The real moon does not look so big as a 239,000-mi disk 12 ft away! The lack of distance contexts for the moon nearly reduces its perception to the law of retinal size, yet not quite. The perceived dummy moon 12 ft away regresses toward the real object to an amount six times what its retinal size would justify, although it is still only one 18-millionth of the size of the real moon. It seems that the great but nearly indeterminate distance of the moon provides just enough context to shift the perceptual size a little bit away from what retinal size alone would give, shifting it in the direction of size constancy, in the direction of regression toward the real object.

Logic or Physiology?

It is an interesting question as to just what is going on when an organism uses the totality of available relevant clues or cues in modeling a perception so that it resembles as nearly as possible the permanent object which is being perceived. From one point of view, the conscious organism seems to be using *clues* to form an inference as to the real nature of the object which is revealed to it through various sense data. From another point of view, the organism seems to be using the various sensory excitations as *cues* to bring a given perception onto the stage of consciousness in accordance with a script in which the stage directions are the integrative properties of the brain. *Clues* and *cues*—both words are used, and they represent two theories of perception, which are often opposed to each other without their being truly incompatible.

Helmholtz recognized this dilemma when, in 1866, he undertook to explain these perceptual phenomena by an appeal to the concept of *unconscious inference* (*unbewusster Schluss*).[10] The perception is essentially, he said, a conclusion formed from evidence by inductive inference. The process of its formation, while like a conscious inference, is actually unconscious. It is normally irresistible and instantaneous, although, Helmholtz thought, it can be unlearned and was therefore probably learned in the first place.

On that last matter Helmholtz certainly made too large a generalization. That coal is black is doubtless learned, and perhaps an artist might unlearn coal's color enough to see it in sunlight as light gray. That is not certain, though. It is possible that the artist sees his light-gray paint as black when he realizes it is coal. Some of the optical illusions break down or diminish under critical inspection, but others persist as inevitably compulsory. No amount of inspection, thought or knowledge will teach you to see the circles of Fig. 3 as anything other than spirals. They are circles though—perfect circles. That they are closed figures appears at once if you start at *a* and follow the circle around until you come back to *a* again.

An illusion is, however, not a good example of unconscious inference, for it involves faulty logic or physiologic, whichever it is. It does not produce a perception that is faithful to its true object. A much better instance of compulsory unconscious inference is the stereoscopic perception of depth.

In stereoscopic vision the evidence for depth or solidity lies in that slight disparity of the two retinal images which is furnished by binocular parallax. Given a few constants of the binocular system, the forms and sizes of the two disparate images, and the assignment of one image to the left eye and the other to the right, and you can figure out geometrically by conscious inference what the dimensions in depth are. The visual mechanism, however, makes this inference instantaneously and unconsciously. If the stereograms are photographs, rich in detail, it is as impossible ever to see the disparity between them, as it is to be aware of disparity in the binocular observation of a solid object. Only in very simple stereoscopic images, such as the outline drawings of geometric figures, does one sometimes see the disparity as doubled lines first, before the doubling disappears and the images pull together into a single solid figure. In the perception the brain reaches a correct inferential conclusion as to the depth of the perceived

[10] H. Helmholtz, *Physiological optics* (1866, tr., Optical Society of America, 1925), vol. III, pp. 1–35; *Die Thatsachen in der Wahrnehmung* (Hirschwald, 1879). Helmholtz seems to have had the idea of unconscious inference as early as 1855. See E. G. Boring, *A history of experimental psychology* (Appleton-Century, 1929), pp. 300–304.

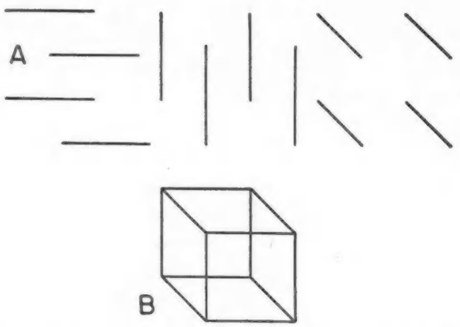

FIG. 4. Dynamics of the visual field. The 12 lines of A, three sets of four lines each, all seen as lying in the plane of the paper, when superposed to make B, form a perceived solid in which the 12 lines no longer appear to lie on the plane of the paper.

object, but the process is no more conscious than is the inference of an electronic computer which calculates almost instantly from relevant data a range and elevation and correctly aims a gun.

It is in fact this electric analogy that answers quite well the question as to whether the integrative process in perceiving is logical or physiological. It is both. There is no contradiction. The inference is as logical and as unconscious as it is when made electrically in a machine.

The Gestalt psychologists — Wertheimer, Köhler and Koffka—have emphasized the notion that perceptual integration in the brain is due to the operation of certain dynamic patterns of "force" that often correct or alter the perceptual form in ways that resemble closely the operation of mechanical or electric field forces.[11] Frequently this process results in what might be called a simplification. Two disparate images, one on each retina, constitute a confusion and make no sense. Put them together and you have both simplicity and sense—a single tridimensional object. Helmholtz would have called that unconscious inference, but the Gestalt psychologists eschew *inference* in favor of *field forces*.

Figure 4 is similar to a figure of Koffka's. Above, at A, you see three sets of four lines each. At B you see the three sets superposed, and the 12 lines arranged in this relation give you the perception of depth. It is practicably impossible to see them any more as lying flat in the plane of the paper, as they did in A. The cube of B insists on appearing solid, although it is reversible and may be seen in either of two perspectives. Is the cube simpler than the 12 lines? Well, there is only one cube, and perhaps one is simpler than 12. At any rate the cube is more sensible than the 12 lines because it is an object. The brain has here integrated an object. For binocular parallax it redintegrates an object.

There is, then, no objection to be raised to the idea that brain properties should operate to establish the conclusions to inferences. Physical properties do in the electric computer. They might in the brain. It takes a brain to do any conscious inferring, and the brain operates always under natural physical law. There is no real contradiction between Helmholtz and the Gestalt psychologists.

It is, moreover, clear that these integrative properties of the brain are both native and acquired, dependent upon both *heredity* and *learning*. The dynamics of stereoscopy and of the perceptions of Figs. 3 and 4 seem almost certainly native neural properties and not acquired. Seeing objects in their true colors and brightnesses, irrespective of the illumination, must be learned. The chromosomes take no account of what colors are in the American flag. It would be a bold man who would assert at present that constancy of perceived shape is either wholly learned or wholly native. We do not know. What then about the correction of retinal size for distance?

There are experiments which show that chickens can learn to choose larger grains of corn and reject smaller, when the difference in size is of the order of only 10 percent. When they have acquired this discrimination, then they will choose remote large grains in preference to small grains near-by, although the retinal images of the remote large grains may be only one-fifth the size

[11] The leaders of the movement called Gestalt psychology are Max Wertheimer (1880–1943), Kurt Koffka (1886–1941) and Wolfgang Köhler (1887–). The movement began in Germany with experiments on perception and with the development of a theory of perception. See reference 1 for Koffka's discussion and the books that give other references to this large literature. A very important contribution is W. Köhler, *Die physischen Gestalten in Ruhe und im stationären Zustand* (Vieweg, 1920); see also his *The place of value in a world of facts* (Liveright, 1938), *Dynamics in psychology* (Liveright, 1940), and Köhler and H. Wallach, "Figural after-effects," *Proc. Am. Phil. Soc.* **88**, 269–357 (1944).

of the retinal images of the near small grains.[12] Still the chickens may have learned to take distance into account.

Children do not do so well as adults, nor young children so well as older, in matching boxes for actual size when the boxes lie on the floor at different distances away.[13] There are individual differences among adults, and anything that tends to distort the perception of distance seems to affect the perception of size, whereas the best judgments of objective size are got from the observers who can estimate the distances most correctly.[14] Such findings suggest that learning plays a role.

On the other hand, it is somewhat implausible that, in learning to compensate for changing size of the retinal image with changing distance, one should acquire a habit for overcompensation (function *A* in Figs. 1 and 2). Overcompensation seems to imply the operation of more basic mechanisms than those that are learned. Still there is no conclusive evidence. One might acquire separately by learning a number of corrective processes which, all working concurrently, would then overshoot the mark. If that conclusion is correct, we are still left with the problem as to why the organism would learn to compensate just a little bit for the small size of the moon's image on the retina. If that correction is learned, it must have been carried over from some other experience.

Actually the decision on this question does not matter. No inherited function is ever quite unaffected by learning, and no learned function is ever able to operate entirely without dependence on what is given it by inheritance. If the perception of size with distance variant depends on both chromosomes and practice, it will be like almost every other psychophysiological function.

Biological Use of Perception

The physiology of perception is uncertain, but its biology is clear. The function of perception is to transform chaotic sense experience into the relative stability of permanent objects, the objects which cause the experience or are implied by the experience, whichever way you like to look at it. An object can be regarded as an as-if theory of experience. Experience would be as it is if there were permanent objects. And the properties of the objects thus become generalizations about experience. So perception, in getting back of experience to the objects, is performing even in primitive man and the animals the same function that science performs in man's civilization. As the purpose of scientific theories is economy of thought, so the purpose of perception is economy of thinking. It picks out and establishes what is permanent and therefore important to the organism for its survival and welfare.

To see a gray as coal is useful. It is to know that this gray will burn and give heat. To see a verdigris as blue is to recognize your country's flag. To see a diamond as square is to recognize the book or the table for what it is, which is important, and to ignore the effect of your own angular relation to it, which, since it changes as you move, is unimportant. To see a distant object with a small retinal image as large and a near object with a large retinal image as small is to get away from the unimportant retinal images to the great importance of the sizes of objects. A chicken by that means gets all the big grains of corn, no matter how far away they are. Perhaps the greatest perceptual achievement of the organism is the way in which it receives on bidimensional retinas optical projections of the tridimensional world, losing, it would seem, all the tridimensionality, and then, taking immediate physiological account of the disparity of binocular parallax and other clues when they are available, instantaneously puts the solid object together again in perception, recovering the tridimensionality of the real object which had seemed irrevocably lost.

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[12] W. Köhler, "Optische Untersuchungen am Schimpansen und am Haushuhn," *Abhandl. preuss. Akad. Wiss., Phys.-math. Klasse*, No. 3, 18–139 (1915); W. Götz, "Experimentelle Untersuchungen zum Problem der Sehgrössenkonstanz beim Haushuhn," *Z. Psychol.* **99**, 247–260 (1926).

[13] F. Beyrl, "Über die Grössenauffassung bei Kindern," *Z. Psychol.* **100**, 344–371 (1926); H. Frank, "Die Sehgrössenkonstanz bei Kindern," *Psychol. Forsch.* **10**, 102–106 (1928).

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Ray Lee Edwards

Recipient of the 1945 Oersted Medal for Notable Contributions to the Teaching of Physics

The American Association of Physics Teachers has made to Professor Ray Lee Edwards, of Miami University, the tenth of its annual awards for notable contributions to the teaching of physics. The addresses of recommendation and of presentation were made by Professor L. W. Taylor, chairman of the Committee on Awards, and Professor R. C. Gibbs, President of the Association, in a ceremony held in McMillin Theatre, Columbia University, on January 25, 1946, during the fifteenth annual meeting.

INTRODUCTORY REMARKS BY PROFESSOR R. C. GIBBS

AT its annual meeting in 1936, the American Association of Physics Teachers made its first award for "notable contribution to the teaching of physics" in the form of a posthumous recognition of the achievements of that unique and stimulating personality, author and teacher, WILLIAM SUDDARDS FRANKLIN.

This award in the form of a bronze medal, known as the *Oersted Medal*, has served to signalize in each of the succeeding eight years the contributions to the advancement of the teaching of physics by one of our fellow members. Thus have we linked with the name of HANS CHRISTIAN OERSTED, himself a teacher and pioneer in the discovery of electromagnetism, a notable roster of American teachers of physics—EDWIN H. HALL, ALEXANDER W. DUFF, BENJAMIN H. BROWN, ROBERT A. MILLIKAN, HENRY CREW, GEORGE W. STEWART, ROLAND R. TILESTON and HOMER L. DODGE.

In this connection, we think of these men not merely as teachers in our field but as sources of inspiration and as stimulators for a host of students by whose records and successes we rightly appraise in no small degree the contributions of those chosen for this honor. In consonance with these thoughts, we turn now to the presentation of the Medalist for 1945 by PROFESSOR LLOYD W. TAYLOR.

ADDRESS OF RECOMMENDATION BY PROFESSOR LLOYD W. TAYLOR

FOR the tenth consecutive year the American Association of Physics Teachers is awarding the Oersted Medal "for notable contributions to the teaching of physics." The committee is this year recommending DOCTOR RAY L. EDWARDS, of Miami University, Oxford, Ohio.

It is a matter of common observation that small colleges produce far more than their share of intellectual leaders. It may be that small colleges, in spite of inadequate equipment, provide the most effective channel through which a really inspiring teacher can make his influence felt. It is certainly true that it is easier dependably to trace the origins of such influence to the proper teacher in a one-man department than in one having several or many members. DOCTOR EDWARDS has spent almost his entire professional life in departments that were actually or for most practical purposes one-man departments. Consequently there is little likelihood of error in attributing to his influence the unusually large proportion of his students who have continued professionally in physics.

Seventy-six have done graduate work in physics and made a career in that field, 32 of them with the doctorate and 24 of them with a master's degree. In addition, more than a score of his recent graduates were in graduate school when we entered the war, a large majority of whom carried their training effectively into war service. Of the 76 well-established career physicists, 25 are listed in the current edition of *American Men of Science*. Perhaps the significance of the latter number will be more evident when we recall that only about one third of the regularly recognized colleges and universities can boast of even a single graduate listed as a physicist in *American Men of Science*. And recall again that since he has run a one-man department for so large a proportion of his professional life, DOCTOR EDWARDS cannot avoid a degree of presumptive responsibility for this record that would be far more questionable if he had had a large department on his hands.

But we are not limited to presumption as a basis for this award, even on the good evidence of the record. The Committee on Awards has in its file some 25 letters from DOCTOR EDWARDS's former students, received in response to requests for information about him. Three refrains appear in these letters with almost monotonous regularity. They are, in order of their frequency: (1) he is filled with enthusiasm for physics and renders that enthusiasm highly infectious; (2) he is terribly exacting, both of himself and of his students; (3) he maintains close contact with his students after graduation; their own fathers could scarcely write and visit more insistently, be more enthusiastic about their successes or more disappointed over their misfortunes.

Let me read a few extracts from these letters.

> His lectures did not sparkle with polished wit: rather they were patient, honest, painstaking descriptions of scientific truth.
>
> Doctor Edwards's teaching technique, as well as his personal example, impressed me with the paramount importance of intellectual honesty—of recognizing the fact that physical processes follow inexorable natural laws and that when an investigator's desires or beliefs influence the outcome of an experiment, the result is neither true nor scientific.

This from the first of his students to go on to the doctorate:

> My most vivid early recollection of Professor Edwards is one of mutual astonishment and embarrassment when, while he was trying to teach us simple harmonic motion, he and we all realized that we did not really know how to use a cosine for projection. The dear old mathematics professor of our freshman year had a trigonometry system all his own and, while we could solve any type of plane or spherical triangle by logarithms, we had never learned the basic ideas of the natural functions. Professor Edwards did not blow up nor blow us up, but we left that lecture with mixed feelings which come back to me every time I see the letters S. H. M.

The following is from one who may have run afoul of DOCTOR EDWARDS in a disciplinary way:

> Doctor Edwards works, eats, sleeps, lives for his better students and for the subject of physics. One cannot include "drinks" or "smokes" on the list, for on these subjects he is not noted for tolerance. However, since he has discovered that a goodly number of his most successful graduates are heavy smokers, he had softened a bit on that subject.

And from another of his students:

> Once he has decided on an individual, he uses all the techniques of a supersalesman to interest the

person in taking the essential basic courses in physics which the university offers.

Against the background of DOCTOR EDWARDS's custom of frequent summer visits to graduate schools, the following observation by one of his students is interesting:

He explained to me his refusal to take a sabbatical leave to which he was entitled by saying: "I can't take a leave, for I have only 13 more years left to teach."

Finally, you might be justified in concluding that here is a man who pours all his enthusiasm and energy into enlisting and training narrow specialists. You would be terribly wrong. Last year DOCTOR EDWARDS was interviewed over the radio by one of his own students. One of the interviewer's questions was:

Doctor Edwards, are you primarily interested in developing physicists or in popularizing physics with students having only an incidental interest in science?

And this—mark it well!—was the answer:

I am very glad you raised that question, George. My primary interests are definitely in popularizing physics with students who are merely seeking a liberal education. That is not accomplished, however, by diluting the subject to the point of insipidity; nor by failing to teach the scientific method, of which physics, properly taught, furnishes the supreme example. I believe it is best accomplished by striving to make physics more worthwhile; that is, making it more satisfying intellecually and giving it a broader base for more general use in wider fields. . . . I believe that physics, properly taught, has as much cultural value as any other liberal arts subject. Some of its most important values are philosophical, and the mechanical, electrical or optical devices with which it directly deals are only incidental.

One of DOCTOR EDWARDS's students, anticipating the present scene, wrote and underscored the sentence: "I know Doctor Edwards will be greatly embarrassed. He's like that." To bring this embarrassment to an end: *Mr. President, I am deeply gratified to present* DOCTOR RAY LEE EDWARDS *in behalf of the Committee on Awards, for receipt of the tenth Oersted Medal, awarded annually by this Association "for notable contributions to the teaching of physics."*

The New Challenge to the Physics Teacher

R. L. EDWARDS
Miami University, Oxford, Ohio

EVERY great enterprise has more or less definite stages of development. There are the stages of pioneering, expansion, consolidation and critical evaluation—recognizing and interpreting increasing obligations to society, adjusting and guiding activities to conform more closely with the needs of society. The science of physics is no exception to the rule. We have had a long pioneering stage, and an important place is reserved for pioneers through generations yet to come. This has been the great tradition of physics, perhaps unparalleled in any other field of endeavor. The primary motivation of the physicist is to know, to do something that no one has known how to accomplish, or even something that has been generally believed to be impossible.

It is not strange that the physicist has sought to avoid arousal of public curiosity in his work. He could not easily forget the fate of Galileo and his followers. Though actual persecution ceased centuries ago, the general public has until recent decades remained hostile toward new ideas. "Darius Green and his Flying Machine" was not written or received in mirth, but in ridicule of a notion regarded as utterly absurd. J. J. Thomson's experiments of less than 50 years ago on the discharge of electricity through rarefied gases were quite unknown to the public. But if he himself had known and had proclaimed that he was establishing a basis for the transmission and reception of speech through space across continents and oceans, we can easily imagine the derision that would have greeted his announcement.

The twentieth century has seen a favorable change in the relation between science and the public. This has come about in part through the achievements of applied science. Yet, in themselves, these achievements would have been

attributed largely to inventive genius. The important new element was the appearance of great scientists who were also gifted interpreters of science—Einstein, Eddington, Jeans, Millikan, A. H. Compton. They have believed that they owe a duty to the public, a belief that most of us now share.

The increasing realization of the responsibility of the physicist to the public was followed by the formation of the American Association of Physics Teachers—one of the founder societies of the American Institute of Physics—and, a little later, by the annual bestowal of the Oersted Medal on someone deemed especially successful in teaching. During the ten years in which the Oersted Medal has been awarded, a notable group of physics teachers has been honored—W. S. Franklin, Edwin Hall, A. Wilmer Duff, Benjamin Brown, R. A. Millikan, Henry Crew, G. W. Stewart, Roland Tileston, Homer L. Dodge. I deeply appreciate being considered worthy of association with such a group.

During the war years the physicist has had to forsake much of his customary work in the field of pure science, but there have been very real compensations. The spectacular success of his efforts in the development and utilization of microwaves such as radar and loran, supersonic devices, magnetic mines, the proximity fuze, rocket and jet propulsion, the large scale release of atomic energy, and many other processes and devices has profoundly altered the opinion formerly held by many that the physicist is an impractical theorist. Indeed the man-on-the-street now recognizes that the battle of the laboratories was the deciding factor in winning the war.

Our enormously increased control over nature holds the promise of large gains in human welfare if our citizens fully realize the necessity of world-wide cooperation. But if mankind refuses to learn this vital lesson, hopes for the future are bleak. We as physics teachers can no longer neglect the field of human relations. We must assist in answering very serious questions. Can the American nation continue to legalize warfare at any level? We have seen how industrial strife further embitters the contestants, impoverishes all of us, and terminates at best in only a truce; that little wars tend to grow into great wars. If we do not solve the economic problems of our organized groups by peaceful methods, can we retain any hope of preserving peace at the international level?

The physicist enters this field with undoubted handicaps, but his habit of examining underlying assumptions is almost as valuable here as in his own field. We should be able to discuss intelligently national and world problems, and perhaps clarify some now all but lost in comparative irrelevancies. Our students and the general public are quite immune to dissertations on ethics and morality; yet our conscious effort to dispel current popular delusions, in which the *rights* of individuals and groups are unduly stressed while their *obligations* are virtually ignored, would help restore to sanity a badly confused nation.

Our responsibilities, our opportunities, are great. Can we meet the challenge?

IF I had a child who wanted to be a teacher I would bid him Godspeed as if he were going to a war. For indeed the war against prejudice, greed and ignorance is eternal, and those who dedicate themselves to it give their lives no less because they may live to see some fraction of the battle won. They are the commandoes of the peace, if peace is to be more than a short armistice.—JAMES HILTON.

A Mechanical Model for the Demonstration of the Franck-Condon Principle

Peter Pringsheim
Ray Control Company, Pasadena 6, California

THE Franck-Condon principle, in its original form, determines the probability of transition between a certain vibrational level of an electronic state of a molecule to another vibrational level of another electronic state, and thus the intensity distribution in a band spectrum. This is a special case of the much more general problem of the conversion of electronic energy into kinetic energy of atoms.

The exhaustive quantum-mechanical treatment of the principle, which is due to Condon, is beyond the understanding of the average undergraduate student. The fundamental idea, as it was at first formulated by Franck, is much more easily visualized; Franck states that the location and the velocity of the relatively heavy atomic nuclei cannot undergo appreciable changes during the exceedingly short time corresponding to the emission and absorption of a light quantum or to the transition of an electron from one quantum state to another. This apparently simple theorem is perhaps the most important among the numerous contributions for which modern physics is indebted to the genius of James Franck. Applied to the spectra of diatomic molecules, the principle explains why a change in the vibrational amplitude of a molecule *must* accompany the transition of the electron from one state to another, if the distance between the nuclei in their position of equilibrium is different for the two electronic states.

The well-known representation of the potential energy E_p of a diatomic molecule by "potential curves" showing E_p *versus* the internuclear distance r greatly facilitates the understanding of Franck's theorem. Since we are interested only in the relative motion of the one atom with respect to the other, the reference system can be chosen so that one of the atoms is at rest on the E_p-axis of the coordinate system; the other atom is vibrating between B' and D' along the horizontal line corresponding to the vibrational state v', as in Fig. 1. At the turning points it is at distances $A'B'$ and $A'D'$ from the first atom, the kinetic energy E_k being zero; at an intermediate point X' it is at the distance $A'X'$ from A', and the kinetic energy $E_k(X')$ now corresponds to the vertical distance from X' to the point Y' on the potential curve, while the potential energy $E_p(X')$ has decreased by an equal amount. When the electronic energy of the molecule is increased by absorption of light, the molecule is transferred, according to the Franck-Condon principle, to the state represented by the upper potential curve *II* along a vertical line (conservation of location), retaining the kinetic energy $\frac{1}{2}mv^2$ which it had at the point X' on curve *I*. The oscillating atom continues thereafter to move until its kinetic energy is spent at point G''; subsequently it oscillates between the points G'' and F''. The probability that the electron will jump while the second atom is in a certain position is (according to Franck's original theory) proportional to the time during which the atom is in the neighborhood of this position; this probability is largest at the turning points B' and D', where the velocity is zero, and smallest at the point of equilibrium $C'(r_0)$, where the energy is all kinetic. Although this last assumption is not strictly valid according to quantum mechanics, it is a useful first approximation. It can be exactly realized in a mechanical model which I have found ex-

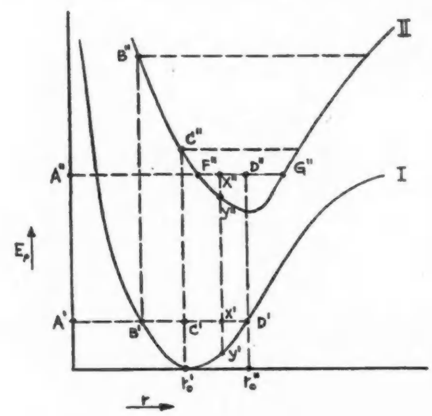

FIG. 1. Potential energy *versus* internuclear distance.

ceedingly helpful in explaining Franck's theorem to students.

The diatomic molecule is represented by an elastic pendulum which vibrates in a vertical plane, and the binding force of which can be altered "instantaneously"; that is, in a time which is very short in comparison with the characteristic period of the oscillator. The latter consists of two horizontal rods, about 20 cm long, connected by springs (Fig. 2). The first rod, made of an insulating material, is rigidly clamped to a stand. The second rod, made of brass, is suspended from the first by means of three spiral springs. The weight of the second rod and the strength of the springs are chosen so that the period of vibration is of the order of magnitude of 1 sec. The two outer springs are permanently attached to both rods, while the central spring can be unhooked from the lower rod. When this is done, the position of equilibrium is altered (from r'_0 to r''_0) and so is the frequency of the pendulum. The two positions of equilibrium correspond to two electronic states of a molecule, in which the binding forces are unequal; for instance, the ground state with a stronger bond and an excited state with a weaker bond between the two atoms. Of course, the two rods represent the two atoms, and the springs represent the bonds.

The lower rod is attached to the central spring by means of a thin wire about 2 cm long, such as is used for fuses. By closing an electric circuit, of

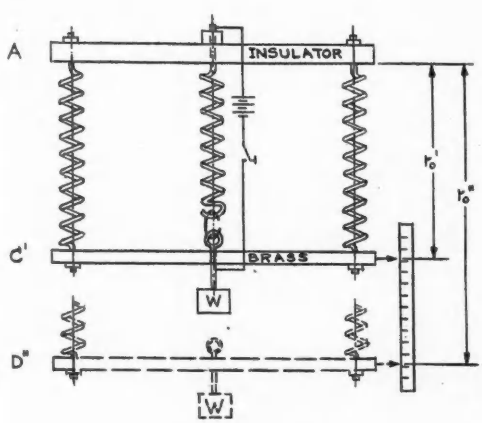

FIG. 2. Mechanical model.

which the central spring forms a part, the fuse is burned almost instantaneously and the system jumps from the ground state to the excited state. In order to be able to repeat the demonstration several times under various initial conditions, it is well to prepare a number of fuses by soldering small copper hooks to the ends of pieces of fuse wire of the right length. If the central spring and the brass rod are equipped with rings or loops into which the copper hooks fit, the fuses can be easily renewed without any loss of time.

If the fuse is burned while the system with the central spring attached is at rest ("non-vibrating ground state"), the pendulum is far from the position of equilibrium corresponding to the excited state: it begins to oscillate with an amplitude equal to the difference $r'_0 - r''_0$; the more the positions of equilibrium in the ground state and the excited state differ, the greater the amplitude of the vibration after the transition from the former to the latter. While the result of this experiment corresponds to expectation and may seem somewhat banal, the inverse experiment is very striking: the pendulum, with the central spring attached, is made to oscillate with such an amplitude that its turning point coincides with the equilibrium position of the excited state. If the electric circuit is closed at the moment at which the oscillator comes to this point, it stops dead, having neither kinetic nor potential energy. For the demonstration of this phenomenon, it is good to mark the heights of the two equilibrium positions or of the "two electronic states" by some sort of pointers attached to the stand. Furthermore, since the oscillation will always be more or less strongly damped, one must start with an amplitude greater than $r''_0 - r'_0$ (distance between the zero positions) and wait for the oscillation at which the turning point is as near as possible to the second equilibrium position before operating the electric switch.

If the central spring is detached at the moment when the excited oscillator passes through its equilibrium position at r'_0, this oscillator has a relatively large kinetic energy in addition to the potential energy with respect to the new equilibrium positions at r''_0 and, therefore, will begin to oscillate around the latter with increased amplitude. It will be much more difficult, how-

ever, to close the electric circuit exactly at this moment, because the velocity of the moving rod is great; this transition has a "small probability." The probability is again large at the upper turning point, but here the potential energy of the oscillator is very large after the fuse has been burned, and the amplitude of the oscillation attains its greatest value under these conditions. Thus, there are two transitions that will occur most frequently from a certain "vibrational level" of the unexcited state: one to the nonvibrating level of the excited state, and one to a level of the excited state in which the vibration occurs with large amplitude. This is exactly what is observed in certain parts of the absorption spectra of diatomic molecules. By imparting to the pendulum in its ground state other initial amplitudes, other less extreme transitions to the excited state can be demonstrated with the model.

Responsibilities of Science Departments in the Preparation of Teachers

A Report of the Committee on the Teaching of Physics in Secondary Schools*

THE problems arising from a deficiency in the number of well-trained science teachers in the secondary schools are as much the concern of the school systems as they are of the teacher training institutions. This committee has given consideration to possible activities on the part of the teachers of physics and physical science.

The opinions expressed by members of the main committee seemed to fall under four heads:

1. The desirability of cooperation between science departments, on the one hand, and the education departments, on the other, in the college program of training secondary school teachers of science.
2. Teacher training as a fundamental responsibility of colleges of liberal arts.
3. Allocation of the curriculum of college students preparing for teaching to the subject fields in which they expect to teach.
4. Certain administrative details.

These are developed in order.

1. *Desirability of cooperation between science and education departments.*—Many excellent teachers of physics and chemistry at the college and university level are not qualified to train prospective secondary school teachers in the art of teaching. The solution of some of the resulting problems would be facilitated by the cooperation of the departments of education with the subject matter departments. The committee seems unanimous in the feeling that a major handicap to effective functioning by science departments in the preparation of teachers is the typically negativistic attitude of these departments toward departments of education. Whether justified or not, this attitude is in itself impeding the discharge of one of the major responsibilities of the sciences, namely, the preparation of teachers in their respective fields.

Any measure that reduces or clears up this aversion (for in many cases it is no less) in individual institutions will facilitate to just that extent the effective preparation of secondary school teachers of science. There is reason to suppose that if individual scientists in colleges would take their problems in this field to chosen colleagues in the department of education, they would receive competent and informed help to a degree that would encourage further cooperative ventures.

In this conviction, the committee definitely suggests such line of action to college teachers of physics. In this way we can make progress in the preparation of science teachers that could not be made in any other way.

* At the meeting of this committee in Chicago on May 5, 1945, a subcommittee was charged with the task of summarizing the informally expressed opinions of committee members on the responsibility of the various science departments in a given college or university jointly with the department of education. This subcommittee, which consisted of C. W. MacLean, T. H. Osgood, J. W. Schneck and L. W. Taylor, *Chairman*, prepared the present report.

2. *Teacher training as a fundamental responsibility of colleges of liberal arts.*—Some 60 percent of secondary school teachers are trained in colleges of liberal arts or in the corresponding colleges of universities. Preference for teachers so trained is still strong with school administrators despite the pressure for placement of graduates of teachers colleges. But this preference is steadily diminishing, partly because many colleges of liberal arts have come to regard teacher training as a merely incidental activity. Believing that teachers can, broadly speaking, be better prepared in colleges of liberal arts, we regret the trend away from them by teachers-in-preparation. *To stem this trend, colleges should regard teacher training, not as a by-product of the liberal arts training, but as a major activity of the college.*

3. *Curriculum for teachers-in-preparation.*—Despite numerous studies in this field, there is little information of an authoritative character available to curriculum-making bodies. Some authoritative body such as the Cooperative Committee on Science Teaching should provide an annotated bibliography with an appropriate summary, suggest such studies as are required to make the job complete and see that their report is made available to curriculum-making bodies of all colleges and universities.

At least four schedules should be prepared: (*a*) the minimum essentials in the curriculum of any student preparing to teach secondary school physics and chemistry or a combined course in the two sciences; (*b*) a complete sample curriculum for such liberal arts students; (*c*) a similar curriculum for students preparing in engineering or technical schools; (*d*) a similar curriculum for students preparing in states that require a five-year course as a preparation for teaching.

4. *Certain administrative details.*—The pattern that has developed in science instruction as conventionally administered to college students should be critically re-evaluated in the light of its appropriateness for the preparation of science teachers. Especially in physics has it seemed necessary to employ unduly expensive equipment, and to only a lesser extent is this true of the other sciences. Few teachers will have the benefit of such apparatus in their early years of teaching in schools. It will be well, therefore, *to acquaint them with simple equipment in addition to the experience they may be given with standard college-grade apparatus, and to demonstrate how effective such simple equipment can be in instruction at the secondary school level.*

The informal projects undertaken in summer "workshops" can be very important elements in teacher preparation and should constitute a part of such preparation.

Finally, *joint participation in the supervision of practice teaching by subject matter departments and the department of education can work to the great advantage of teachers-in-preparation.* A most valuable element in the training of teachers of physics and chemistry could be to allow them to assist in the setting up of lecture demonstrations and, in cases where it seems justifiable, to deliver at least portions of such lectures themselves.

Counsellors from colleges can visit schools to great advantage, receiving information on local problems and giving help in their solution. Such counsellors, versed in subject matter and grounded in educational theory, with experience in secondary education but in the employ of colleges or universities, are not easy to find. At least one university has experimented in this activity to the great advantage of all concerned.

K. LARK-HOROVITZ, *Purdue University;* C. W. MACLEAN, *Westinghouse Electric Corporation;* T. S. OSGOOD, *Michigan State College;* G. E. O. PETERSON, *Schurz High School* (Chicago); F. H. PUMPHREY, *General Electric Company;* J. W. SCHNECK, *Riverside High School* (Milwaukee); R. J. STEPHENSON, *College of Wooster;* L. W. TAYLOR, *Oberlin College;* G. W. WARNER, *Wilson Junior College.*

NO man will ever comprehend the real secret of the difference between the ancient world and our present time, unless he has learned to see the difference which the late development of physical science has made between the thought of this day and the thought of that (of ancient Greece), and he will never see that difference, unless he has some practical insight into some branches of physical science.—THOMAS HENRY HUXLEY.

Demonstrations with a Large, Low Speed Gyroscope

Harold K. Schilling
The Pennsylvania State College, State College, Pennsylvania

THREE serious criticisms can be leveled against lecture-demonstration gyroscopes commonly available on the market. (1) They are too small for use before large classes. (2) Component parts are not marked distinctively so that students can distinguish and follow them readily during experiments. (3) Usually they require high speed spins, with the result that students cannot perceive directly in which sense the spin occurs, or, indeed, that there is any spin at all.

Doubtless one reason for the popularity of the bicycle wheel used as a gyroscope is that it is large. However, as usually sold it does not meet either objection (2) or, for many experiments, objection (3). Moreover, quantitative experiments are not easily possible because, when it is both supported and manipulated by the demonstrator, it is often not at all apparent to the class whether large or small torques are used or in which direction they are applied.

The teaching effectiveness of the bicycle-wheel gyroscope can be greatly enhanced by mounting it in gimbals as illustrated in Fig. 1. The wheel is placed in a yoke Y, which is free to rotate about a horizontal axis yy'. This in turn is supported by a structure S resting on a turntable such as is commonly used in conjunction with these gyroscopes; Y and S are made of wood and, together with the wheel, are light enough to be lifted and moved by hand.

The wheel is not permanently mounted since it is needed for other experiments. On each side of the yoke is a deep notch into which the handles H of the wheel can be locked by means of hardwood wedges W. These slide in grooves, and are slotted so they can be held in place by bolts and wing nuts. From the center of the bottom of the supporting structure S there protrudes a short rod which fits the socket in the center of the turntable. The block B is slotted so it can be moved up and down and locked in any desired position by means of wing nuts on bolts. Its purpose is to limit the rotation of the yoke when that is desired. There are hooks h on opposite ends of support S, and a screw eye in each end of handles H. At F on the yoke there are flush plates into which may be screwed 10-mm rods for holding weights. For some experiments these weights are used to raise or lower the center of gravity of the yoke and wheel and thus produce stable or unstable equilibrium.

The apparatus is painted in contrasting colors. One half of the wheel, including the spokes, is painted black, and the other half white. This enables one to observe more easily both the sense and approximate speed of rotation. One side of the yoke Y and the corresponding wheel handle are red, while the opposite side and handle are yellow. One-half of the support S, including the end of axle y, is green, the other blue. The three axes are therefore readily recognized and may be referred to conveniently as the vertical axis, the green-blue axis, which is always horizontal, and the yellow-red axis of spin.

This sort of color scheme is helpful also in simplifying language difficulties during the demonstration. It minimizes the use of the ambiguous "this" or "that." One can speak, for instance, of the spin vector as being directed along the yellow-red axis—say from the red toward the yellow—and can thus avoid the awkward circumlocutions so often necessary when things do not have distinctive names.

The following demonstrations can be carried out easily and quickly. Since the rate of spin required is not large, the wheel can be stopped readily at any time— to interrupt the sequence of experiments in order to repeat a particular one or to change its physical conditions.

1. Direction of Precession

With the three axes initially mutually perpendicular, and the wheel spinning, a hooked weight hung from one end of the yellow-red axis causes precession of the green-blue axis about the vertical. Any weight from 100 gm to 1 kg is suitable. The wheel need be turned only by hand.

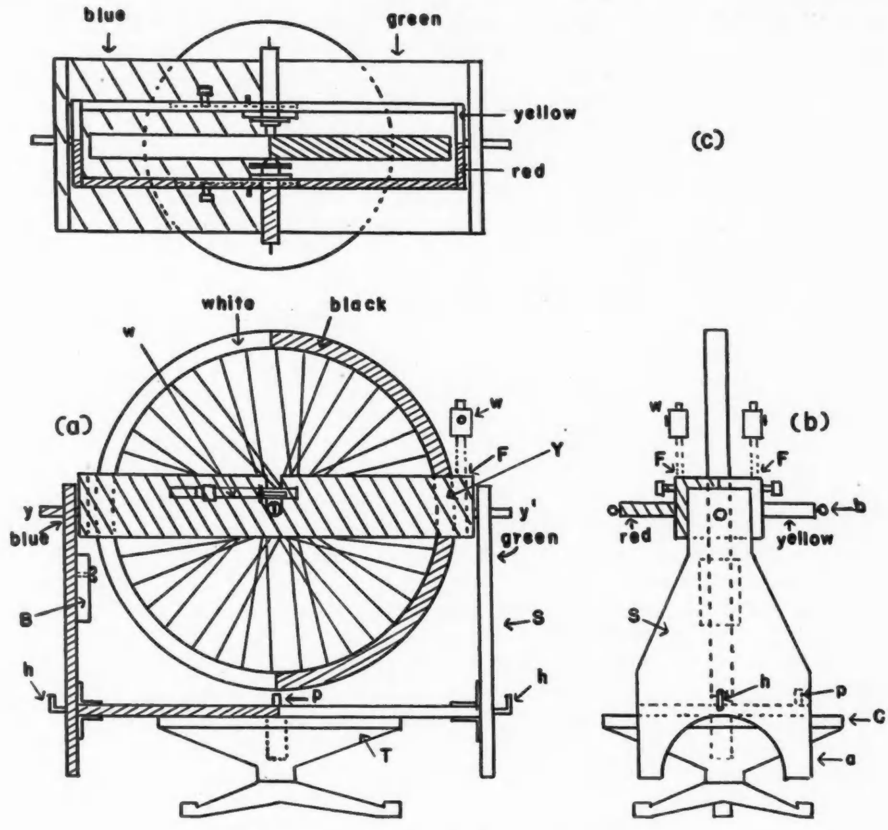

FIG. 1. (a), front elevation; (b), end elevation; (c), plan.

The effect of an opposite torque and of opposite spin are then demonstrated.

Precession of the yellow-red axis about the green-blue axis is achieved by pushing sidewise against the supporting structure S, say at a in Fig. 1(b). It may be reversed, of course, by an oppositely directed torque, or by reversing the spin.

2. Magnitude of Precession

It can be shown with fair accuracy that the rate of precession ω_p is directly proportional to the torque T producing it and inversely proportional to the rotational velocity of spin ω_s. The precessional period can be measured conveniently with a stopclock. For a given spin the period produced by a 1-kg weight is half that obtained with 0.5 kg, and one-fifth that obtained with 200 gm, with errors not greater than a few percent, unless the weight is less than 200 gm or the spin is very slow.

Likewise it can be shown that when the spin velocity in one case is twice that in another, other things being equal, the precessional velocities are in the ratio of one to two. This demonstration is somewhat more difficult than the former one, but can be achieved with some practice.

3. Retarding or Accelerating Precession

These experiments can be described best by referring to Fig. 1(b). Suppose the wheel is spinning in such a way that, when a weight is hung at b, the end of S facing the reader pre-

cesses toward the right. When this precession is opposed or retarded by friction produced against the protruding rim of the turntable at c, the result is that the axis of spin, and the weight at b, dips more rapidly than when the precession is unimpeded. This phenomenon may be explained by referring to previous experiments and pointing out that the torque opposing the precession of the green-blue axis produces precession of the yellow-red axis in the direction of the dip. Conversely, accelerating the precession causes that axis to rise.

Consider next the case of precession about the horizontal, green-blue, axis. The necessary steady torque is exerted by a string attached to the hook at h, Fig. 1(b), which passes horizontally to the left and over a ball-bearing pulley, say 6 ft away, and from which is hung a 200-gm weight [see also Fig. 2(a)]. If the wheel is spinning in the same sense as in the preceding experiment, the right-hand end of the spin axis precesses downward. This precession may be accelerated by hanging a weight, say 100 gm, on that end or may be opposed by a weight on the other end. In the former case the support S will move less rapidly toward the left, and in the other case more rapidly. For a particular, properly chosen weight, S remains fixed until the angle of dip has become nearly 90°.

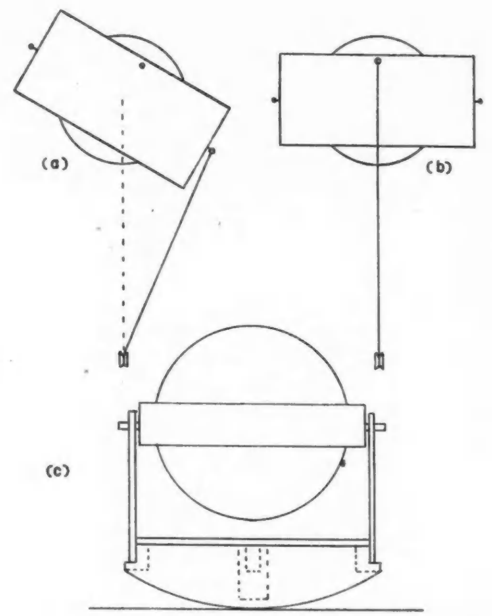

FIG. 2. (a), arrangement for experiments **3** and **4**; (b), arrangement for experiment **5**; (c), illustrating use of rocker.

4. The Damping of Oscillation

Here again we use the arrangement illustrated by Fig. 2(a). If the support S is released while the wheel is at rest, the pull of the cord will cause it to oscillate, about the vertical axis and on either side of the line joining the pulley and the vertical axis, through at least ten cycles before it comes to rest. However, when the wheel is spinning the oscillation is damped out in a quarter of a cycle—owing primarily to friction at the axis of precession.

It is important for the student to understand that this damping is possible only when the gyroscope is free to precess and when friction opposes the precession. The first of these two requirements can easily be demonstrated with this apparatus. Precession about the horizontal, green-blue axis may be prevented by raising the block B, Fig. 1(a), and clamping it tightly against the bottom of the yoke, thus fixing the red-yellow axis of spin in the horizontal plane. There results then only underdamped oscillation, even though the wheel may have a high spin velocity.[1]

5. Stabilization When There Are Two Degrees of Instability

The preceding group of experiments illustrates the basic principles underlying the action of ship stabilizers. Now we consider the kind of stabilizer required for, say, a monorail car. Here two degrees of instability are required. Instability with respect to the green-blue axis is accomplished by mounting weights w (each about 400 gm) above the center of gravity, on the yoke, as shown at F in Fig. 1, (a) and (b). With respect to the vertical axis instability is achieved by looping a string over peg p, and passing it over a pulley to a 1-kg weight, the

[1] The present apparatus was built during the war when materials were hard to get. The next model will have ball bearings at y and y', Fig. 1(a), and will have variable friction control for demonstrating the role of friction.

peg being on the farther side of S with respect to the pulley, as suggested by Fig. 2(b). When, under these conditions, the wheel is put into rotation the system as a whole is seen to be stabilized despite the fact that individually the yoke with the wheel and the support were initially in unstable equilibrium. When released, both support S and the yoke Y, with the wheel, oscillate with increasing amplitude about their initial orientations until one of the two degrees of instability disappears and the stability of the system can therefore no longer be maintained. This occurs either when, in the course of oscillation, the center of gravity of yoke Y and wheel falls below the green-blue axis, or when the oscillating support S collides with an obstacle or the string.

If the block B is clamped against the yoke so it cannot turn, thus eliminating one degree of instability from the very start, there is no stabilization even though the wheel is spinning.

It is instructive also to show the contrasting phenomena resulting when the weights w are initially below rather than above the axis, or when the support is turned through 180° to begin with, so that the peg p is between the pulley and the vertical axis. In both cases there is initial stability with respect to one axis and instability with respect to the other.

The great advantages of the apparatus for the demonstration of this particular experiment —one with which students have much difficulty —are that it proceeds slowly enough so the lecturer can point out to the class just what happens step by step, and that he can change the physical conditions so easily.

6. Effect of Impulsive Torques, When There Is No Instability; Inertia; Restoring Force

We start with neutral equilibrium relative to each axis and with all axes perpendicular to one another. The wheel is then given a spin velocity. When, thereafter, the support S is given a sharp blow so as to initiate rotation about the vertical axis, both this axis and the yellow-red axis oscillate about their original positions with decreasing amplitude until friction stops them. Of course, the same oscillation can be produced by giving one end of the red-yellow axis a blow.

This illustrates rather interestingly the interplay of inertia and the restoring force resulting from precession. It should be emphasized that again the demonstration is not complete until it is shown that there can be no restoring force unless there is precession. Thus if, by means of the block B, the red-yellow axis is fixed in the horizontal plane, precluding precession, there is no stability, as evidenced by the fact that the blow against S simply sets up rotation rather than oscillation about the vertical axis. That there is inertia is apparent, but no restoring force appears.

Likewise, if the support S is removed from the rotating platform and clamped to the table so it cannot precess when the red-yellow axis is given a blow, the yoke simply rotates on the green-blue axis, just as it would if the wheel were not spinning in the first place.

Nor is this experiment complete until it is shown what happens when one does *not* start with neutral equilibrium. For this purpose the weights w are again attached to the yoke, this time below the center of gravity of the wheel. This creates stability relative to the green-blue axis. Obviously, when S or H is given a blow under these conditions, a restoring force due to gravity enters the picture. If a student really understands earlier experiments, he (and the reader) should be able to predict the over-all result.

7. Other Demonstrations

Teachers will, of course, think of many other experiments to be made with such an apparatus. Hence only one further suggestion is made here in conclusion. The support S may be placed on a "rocker" as illustrated by Fig. 2(c). The top of the rocker should have a socket into which to fit the pipe protruding from the bottom of S. Also, the supports may be placed on the rocker in a transverse position. These setups permit experiments which, though they may not introduce new basic principles, do nevertheless allow application of these principles to new situations. The whole subject is so difficult that different approaches are not only desirable, but seem necessary for mastery.

The writer is indebted to his colleagues Dr. F. Raymond Smith and Dr. M. Parker Givens for reading this paper critically and making valuable suggestions before publication.

Two Experiments to Demonstrate the Inertial Significance of Mass and the Conservation of Momentum

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THE two experiments described in this paper are designed for use in an elementary physics course either as lecture demonstrations or as laboratory exercises. Both methods yield relatively accurate measurements of mass. There are, of course, other inertial arrangements for comparing masses.[1–3]

The two basic experimental facts to be illustrated are the following:

(i) Two particles in an isolated interaction are accelerated in opposite directions with respect to an inertial system, and the ratio of the changes in their velocities during any interval of time is constant. Designating one particle as a standard and successively comparing all others with it, one gets

$$\frac{\Delta \mathbf{v}_s}{\Delta \mathbf{v}_1} = -k_1, \quad \frac{\Delta \mathbf{v}_s}{\Delta \mathbf{v}_2} = -k_2, \quad \ldots, \quad \frac{\Delta \mathbf{v}_s}{\Delta \mathbf{v}_x} = -k_x, \tag{1}$$

where each k is a positive numeric.

(ii) When any two particles which have been compared with the standard are compared with each other, the foregoing numerics are found to have far-reaching significance for which there is no *a priori* expectation from Eq. (1): in an isolated interaction between any two particles x and y, it is observed that

$$\Delta \mathbf{v}_x / \Delta \mathbf{v}_y = -k_y / k_x. \tag{2}$$

The property of matter thus revealed is called inertia. The description of this property in Eqs. (1) and (2) constitutes the principle of inertia. The physical quantity *mass* is introduced as a quantitative measure of inertia—a third fundamental quantity and physical dimension. By definition, according to Mach, a specific magnitude, or unit of mass, is assigned to the standard, and the mass of particle x is taken to be k_x times that of the standard; thus, $m_x = k_x m_s$. Substituting m_x/m_s for k_x in Eqs. (1) and (2), one gets the law of conservation of linear momentum:

$$\Delta \mathbf{v}_x / \Delta \mathbf{v}_y = -m_y / m_x,$$

or

$$m_x \mathbf{v}_x + m_y \mathbf{v}_y = m_x \mathbf{u}_x + m_y \mathbf{u}_y.$$

The demonstrations described here represent a special case of interaction in which two particles start from rest. In this special case, $v_x/v_y = m_y/m_x$ at each instant, from which it follows that

$$s_x/s_y = m_y/m_x,$$

where s_x and s_y are the respective distances traveled by the two particles during the same interval of time; the position of the center of mass of the two particles is unaffected by the mutual action between them.

Momentum Car Experiment

The experimental arrangement is shown in Fig. 1. A compressed spring propels the two cars L and R apart along a smooth horizontal surface when the cord that holds them together is burned. A 4-in. length of hard aluminum tubing, $\frac{3}{16}$ in. in outside diameter, slides through close fitting bushings in the ends of a tube of larger diameter hinged on a horizontal axis at the center of car R. The aluminum tubing extends through the spring and into a socket hinged on a horizontal axis at the end of car L. It has four slits milled lengthwise in it near its free end so that its walls can be bulged slightly outward. The tubing serves a twofold purpose. It guides the cars along a straight-line path, and it brings them to a dead stop when its fluted end enters the bushing adjacent to that end.

The cars that I have used are adaptations of the three-wheel momentum cars made by the Central Scientific Company. The flanges on the

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[1] H. B. Lemon and M. Ference, Jr., *Analytical experimental physics* (Univ. of Chicago Press, 1943), pp. 30–31. Motion pictures have been taken of two cars propelled apart by a spring. Relative masses are determined from measurements of distance on different frames of the film.

[2] D. Roller, "Mass and force as kinetical concepts," *Am. J. Physics* **4**, 99 (1936).

[3] W. Schriever, "A new inertial balance and operational definition of mass," *Am. J. Physics* **5**, 202 (1937).

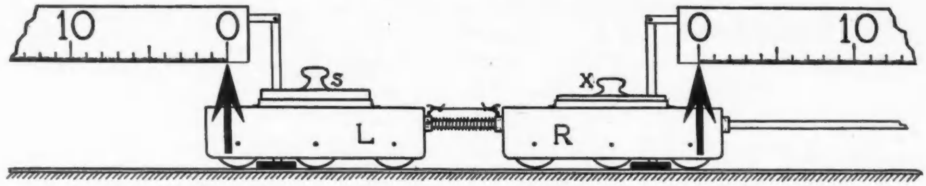

FIG. 1. Momentum-car arrangement for comparing masses. The essential feature is that the two cars are brought to rest after being propelled apart through distances in the inverse ratio of their masses.

wheels were ground off to protect the runway, which consists of a planed and polished strip of seasoned wood with cleats running lengthwise along its underside to prevent warping. The mass of each car, including its accessories (the spring and the aluminum rod in the case of car L), was made equal to 500 gm. A 500-gm lead standard was constructed in the form of a squat cylinder with a knob to make it look something like a standard. A shallow cylindrical receptacle was added to the top of each car to hold either the standard or a similar 300-gm cylinder to be used as a body of unknown mass. The center of mass should be kept low to prevent the cars from jumping into the air when the spring is released.

A position indicator was added to each car in the form of a thin metal strip extending in a vertical plane across the width of the car, with a prominent arrow on its front edge to attract student attention; it is convenient to have arrows on both edges of the indicators so that either side of the car combination may be placed toward the front. The position of each indicator along its scale may be read to a fraction of a centimeter throughout a fairly large lecture room and to a fraction of a millimeter by the lecturer. The cars, held together by a cord, are placed so that the indicator on one car coincides with the zero of its scale; then, by means of a rack and pinion not shown in Fig. 1, the other scale is moved along its supporting bar a few millimeters one way or the other to effect a similar coincidence. If cords of appropriate length with a small loop on each end are prepared in advance, only a few seconds are needed to make ready for each trial.

When the apparatus is first set up for a lecture, the distances traveled by the empty cars will probably not be exactly equal. The motion is influenced by two factors: one end of the runway may be a little higher than the other end; and the cars may differ slightly with respect to friction even though the pivot bearings have been adjusted so that each has about the same degree of tightness. The resulting effects on the motion may be either compensatory or additive; if the cars are turned end for end on the table, one is likely to find that the distances through which they move are more nearly equal in one orientation than in the other. One end of the runway should be shimmed until the empty cars do move through equal distances. Accurate adjustment takes only a few minutes. It is interesting to observe that a change in level of the plane which is hardly discernible with a spirit level produces an unmistakable change in the ratio of the distances traveled by the cars.

Little time is needed to carry out a complete demonstration in four steps as follows:

(1) It is shown that the empty cars are propelled through equal distances. The students are led to infer that the ratio of the distances traveled would be the same in a mutual action between the cars in space, where they would be effectively isolated from all other bodies. It is concluded that the two cars have equal masses, m_c.

(2) With the 500-gm standard mass m_s in one car, it is shown that the empty car travels twice as far as the loaded car. It is concluded that $m_c+m_s=2m_c$, whence $m_c=m_s=500$ gm.

(3) With the body of unknown mass m_x in one car, it is shown that the empty car travels 1.6 times as far as the loaded car. It is concluded that $m_x=0.6m_c=300$ gm.

(4) The standard mass having been placed in one car and the body of mass m_x in the other car, the students are asked which of the two cars should travel the greater distance, and what should be the ratio of these distances. Comparing masses, 1000 gm on one side and 800 gm on the other side, it is predicted that the car with the smaller total mass should travel the greater distance in the ratio 10/8, or 1.25.

A representative set of measurements is re-

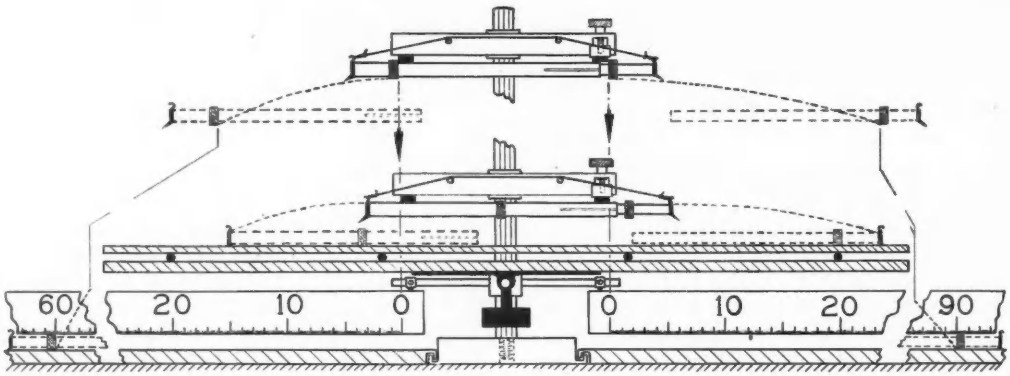

FIG. 2. Projectile arrangement for comparing the masses of telescoping tubes (see upper position of tubes and supporting bracket) and for demonstrating conservation of momentum (see lower position, from which the tubes are projected onto a thin light board with full freedom of motion in the plane of motion of the tubes).

produced in Table I. It may be seen that the results depend to a slight extent upon which car a given body was placed in. This discrepancy is attributed to the fact that the cars did not run on a perfect plane; using a spirit level, I found that the runway was not entirely free from hump and hollow. Even so, the results are rather good. And an important point is this: the variation in repeated trials with a given combination is small enough so that, in a lecture, one may rely upon a single trial for each of the four steps with full confidence that the demonstration will not be spoiled by an unexpected result.

Projectile Experiment

In this experiment, the effect of friction is largely eliminated. Two telescoping tubes are propelled apart in a horizontal direction by a spring as they fall through the air. During their brief mutual action, each tube has two independent accelerations: a horizontal acceleration due to the interaction between the tubes and a vertical acceleration due to gravity. The independence of these two accelerations is a good illustration of the principle of superposition. One may readily infer that the accelerations of the tubes due to their mutual action would be the same in remote space, where there is no acceleration due to gravity.

In Fig. 2, the bracket which supports the tubes is shown clamped at two different heights to a $\frac{3}{4}$-in. table rod. The dimensions may be gauged from the centimeter scales shown in the figure. The tubes, after being tied together, are held against the underside of two short bars extending out toward the front of the bracket, and the cord is stretched over two supporting pins as shown; the level may be accurately adjusted by means of a leveling screw on one of the bars. The cord may be severed almost instantaneously by pressing upward on it with a razor blade in a suitable holder. The outer end of each tube has a flange containing five sharp prongs inserted around its lower half at an angle of about 45° with the axis of the tube; only one such prong is indicated in Fig. 2. These prongs bring the tubes to a dead stop when they impinge upon a softwood board. A short pin in the inner tube fitting a thin slot in the outer tube prevents any rotation of the one tube with respect to the other, about a central axis, as they fly apart. Each tube has a position indicator in the form

TABLE I. Comparison of masses by the momentum car method.

$\frac{m_R}{m_L}$	$\frac{m_R+m_s}{m_L}$	$\frac{m_L+m_s}{m_R}$	$\frac{m_R+m_x}{m_L}$	$\frac{m_L+m_x}{m_R}$	$\frac{m_R+m_s}{m_L+m_x}$	$\frac{m_L+m_s}{m_R+m_x}$
1.000	1.99	2.01	1.58	1.59	1.25	1.27
1.001	1.98	2.00	1.57	1.58	1.25	1.28
1.000	1.97	2.00	1.58	1.57	1.25	1.27
1.000	1.99	2.02	1.58	1.59	1.24	1.26
1.000	1.97	1.99	1.57	1.59	1.25	1.27
1.002	1.98	2.00	1.58	1.59	1.24	1.27
Beam balance values						
1.000	2.000		1.600		1.250	

of a sliding collar, drawn with double cross-hatching in Fig. 2.

Comparison of masses.—The indicator on each tube is placed so that its inner periphery stands directly above the zero of its scale, as shown in the upper position of Fig. 2. The tubes are projected onto boards which may be slid directly under the respective scales for an accurate reading of the horizontal displacements. The time of fall may be varied by changing the height of the bracket.

Table II contains a typical set of results obtained with three telescoping tubes A, B and C; C fits inside B, and B inside A, so that A and C may each be compared with B, and B plus C with A. Tube A contains a spring fastened permanently in its flange end. Tube B contains a spring which is held in its flange end for a comparison with C, and in the opposite end when B and C are held together as a unit.

Conservation of linear momentum involving a third body.—A platform, not needed in the comparison of masses, is clamped to the table rod as shown in the lower position of Fig. 2. The tubes are projected from a height of about 1 in. onto a thin light board resting on four dowel pins which are free to roll with very little friction along the platform; thus horizontal translation of the board is practically unimpeded. The freedom of motion may be demonstrated in striking fashion if the board is placed far enough to one side so that only one tube falls on it.

Here is a good chance to test the student's understanding of mass and momentum as gained from the preceding demonstrations. What will happen when the tubes impinge upon the board? Will the left-hand tube carry the board and the other tube along with it toward the left? Or will the right-hand tube carry the board and the other tube along with it toward the right? Or will there be any translation of the board?

The initial momentum of the board and the tubes is zero, and their total momentum in a horizontal direction remains zero because none of the three bodies is subjected to any action in a horizontal direction except that which they exert on one another. No motion whatever is imparted to the board.

Conservation of angular momentum.—The platform and board are balanced and left free to rotate about the rod which supports them. The center of gravity should be low enough so that, when the tubes are added to the board, the equilibrium is not unstable; hence a counterweight has been indicated in solid black. (In actual construction, the supporting rod is best

TABLE II. Comparison of masses by the projectile method. The two columns for each mass ratio show the values obtained in alternate measurements in which the body that was first projected toward the right was next projected toward the left.

m_A/m_B		m_C/m_B		m_A/m_{B+C}	
1.490	1.483	0.504	0.487	1.012	1.011
1.495	1.477	0.509	0.498	1.030	1.021
1.496	1.482	0.502	0.488	0.993	1.001
1.495	1.473	0.500	0.490	0.979	1.000
1.480	1.476	0.504	0.487	0.991	1.001
Mean,	1.485	Mean,	0.497	Mean,	1.004*
		Beam balance values			
1.491		0.492		1.000	

* *Check calculation:* taking the mass of B equal to 1.000 unit, one gets from the inertia measurements in the first two columns,

$$m_A/m_{B+C} = 1.485/1.497 = 0.992.$$

placed on the upper side of the platform at about the level of the dowel pins so that little or no counterweight is needed; the platform has been drawn above the supporting rod in Fig. 2 so as not to obstruct the view of the upper part of the scale and its support.) The indicator on the outer tube is placed at the center of mass of the two tubes, as determined by suspending them from a single pin. They are then placed on their supporting bracket with their center of mass directly above the axis of rotation. Will there be any motion of the board when the tubes fall on it?

The angular momentum of the tubes before impact is zero, and so is that of the board and platform. The total angular momentum remains zero. No matter what the relative masses of the tubes may be, the impact produces neither rotation nor translation.

I wish to express my appreciation to Mr. George Kayser for his skilful construction of the apparatus, his cheerful cooperation in making changes during its development, and his assistance in taking many sets of measurements.

The Dynamics of a Roll of Tape

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IN a note entitled "Nonconservation of energy —a paradox," L. L. Pockman has drawn attention to a mechanical question of some interest beyond the particular situation described.[1] It is pointed out that if a solid roll of thin, flexible ribbon is allowed to roll without slipping down an incline from rest, unwinding as it goes, the gravitational potential energy of the ribbon after unwinding completely will be less than the initial potential energy. The question is raised: "What has become of the difference?"

In seeking to understand the unrolling process, one is prompted by the occurrence of an energy loss to look for impulsive forces, and the impact on the plane of each element of the tape suggests itself as a possible cause of energy dissipation. Yet, if the roll may be regarded as a cylinder of finite radius, no such impacts occur, as a simple consideration shows. Figure 1 represents a portion of the edge of a rolling cylinder near its point of contact with the plane. Any point P has, with respect to fixed axes, the forward speed v of the entire disk and, simultaneously, the rim velocity, which is of the same magnitude and directed tangentially. The resultant is w, and it is evident that as P approaches contact with the plane, the magnitude of w approaches zero ("the contact point of a wheel is momentarily standing still"). No dissipation of energy is to be sought on this account, provided that both v and the radius r remain finite.

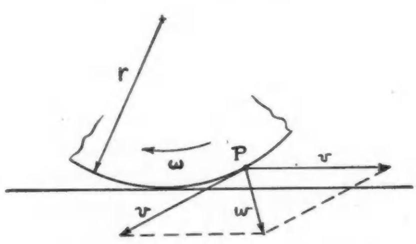

FIG. 1. Motion of a point on the edge of the roll.

A detailed analysis is therefore appropriate, and the energy method proves to be most direct. Let l represent the total length of the tape, R the initial radius of the roll, and r the radius after unrolling a distance x. If the uniform thickness of the tape is e, the area of the face of the fully-wound roll $\pi R^2 = le$, and the face area after rolling a distance x is $\pi r^2 = (l-x)e$. Combining these two equations, we obtain

$$r = R(1-u)^{\frac{1}{2}}, \qquad (1)$$

where $u[=x/l]$ represents the fraction of the total length unwound. Further, since the mass per unit length of the tape is constant,

$$m = M(1-u), \qquad (2)$$

where m is the mass remaining on the roll after a length x has been unwound, and M is the total mass of the tape.

The path of the center of gravity of the roll, referred to the plane on which it travels, is given by Eq. (1), and is seen to be a parabola symmetric about the axis of x (or u) and with its vertex at $x=l$ (Fig. 2). It is then evident that, when placed at rest on a horizontal plane, the roll will unwind of its own accord; and without loss of generality we may consider this situation instead of the more cumbersome one where the roll is placed on an incline.

The potential energy of the system may be taken to be zero in the initial position. Then the potential energy E_p after rolling a distance x is

$$E_p = -mg(R-r) - (M-m)gR, \qquad (3)$$

the first term referring to the part remaining in

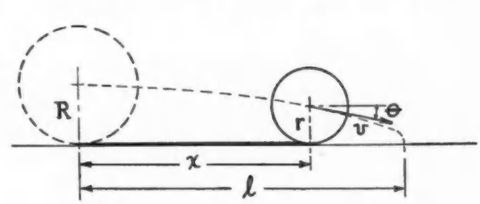

FIG. 2. Geometry of a solid roll.

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[1] *Am. J. Physics* 9, 51 (1941).

the roll, the second to that already lying flat on the plane. Substitution of the values of r and m from Eqs. (1) and (2) into Eq. (3) reduces the last to

$$E_p = -MgR[1-(1-u)^{\frac{3}{2}}]. \tag{4}$$

As in the familiar problem of a rolling solid disk, the kinetic energy E_k at any instant is composed of translational and rotational contributions:

$$E_k = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2. \tag{5}$$

Here $v[=ds/dt]$ is the instantaneous linear velocity of the center of gravity of the roll; $I[=\frac{1}{2}mr^2]$, the moment of inertia of the roll about the axis through this point; and $\omega[=v\cos\theta/r]$, its angular speed of rotation. From Eq. (1),

$$\tan\theta = dr/dx = -R/2l(1-u)^{\frac{1}{2}}$$

and

$$\cos^2\theta = (1+\tan^2\theta)^{-1} = \frac{4l^2(1-u)}{R^2+4l^2(1-u)}.$$

Substitution in Eq. (5) of the foregoing values of I, ω and θ, and of the value of m from Eq. (2), yields

$$E_k = \tfrac{1}{2}Mv^2(1-u)\frac{R^2+6l^2(1-u)}{R^2+4l^2(1-u)}. \tag{6}$$

Inasmuch as the initial potential energy was taken to be zero, the statement of the energy principle becomes

$$E_p + E_k = 0, \tag{7}$$

or, using Eqs. (4) and (7) and solving for v^2,

$$v^2 = 2gR\frac{[1-(1-u)^{\frac{3}{2}}][R^2+4l^2(1-u)]}{[1-u][R^2+6l^2(1-u)]}. \tag{8}$$

The same result can of course be obtained by using the Lagrange equation; the latter, however, leads to a second-order differential equation which must then be integrated to obtain Eq. (8).

Examination of Eq. (8) reveals the cause of the energy loss. At the start, $u=0$, and Eq. (8) then gives $v=0$, as must be the case. At the end of the unrolling process, however, $x=l$, and so $u=1$. When u is allowed to approach unity in Eq. (8), v becomes infinite. Thus, *as the process nears its end, the last bit of tape moves with a speed approaching infinity*. This, in turn, implies dissipation of energy. It is to be noted also that the end of the tape is moving in a direction normal to the plane at the end of the process.

The net loss of energy is equal to the difference between the potential energies at start and finish, and amounts to $\Delta E = MgR$.

Further reduction of Eq. (8) is not feasible, since this leads to a hyperelliptic integral. Instead, a special solution of the problem which lends itself to numerical evaluation will be sketched. Consider the case of a thin tape wound on a massless cylindrical bobbin, so that the radius remains constant. The bobbin is assumed to roll from rest down a plane having an angle of inclination α (Fig. 3).

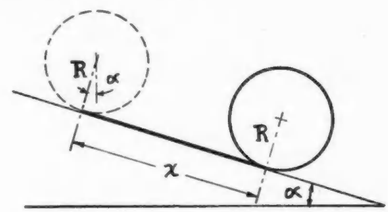

FIG. 3. Roll of constant radius on an incline.

Using a notation similar to the previous, we now have for the potential energy after rolling a distance x along the slope,

$$E_p = -mgx\sin\alpha - (M-m)g(R\cos\alpha + \tfrac{1}{2}x\sin\alpha). \tag{9}$$

The kinetic energy becomes simply

$$E_k = m\dot{x}^2. \tag{10}$$

The energy equation, $E_p+E_k=0$, yields, on substituting $u=x/l$ and using Eq. (2),

$$\dot{u}^2 = A^2\frac{u(B-u)}{1-u}, \tag{11}$$

where $A^2 = g\sin\alpha/2l$, and $B=2(r\cot\alpha/l+1)$. Again it is seen that the velocity $\dot{u}$ becomes infinite at the end of the process, where $u=1$.

It is possible here to calculate the complete time of unwinding, which has a finite value. According to Eq. (11) this is given by

$$T = \frac{1}{A}\int_0^1 \left[\frac{1-u}{u(B-u)}\right]^{\frac{1}{2}} du. \tag{12}$$

Upon rationalization, the integral is seen to be elliptic. Substitution of t^2 for u yields

$$T=\frac{2k}{A}\int_0^1 \frac{1-t^2}{[(1-t^2)(1-k^2t^2)]^{\frac{1}{2}}}dt, \tag{13}$$

with $k^2=(1/B)<1$. Equation (13) may be written in the form

$$T=\frac{2}{kA}[\mathbf{E}-(1-k^2)\mathbf{K}], \tag{14}$$

where

$$\mathbf{K}=\int_0^1 dt/[(1-t^2)(1-k^2t^2)]^{\frac{1}{2}},$$

and

$$\mathbf{E}=\int_0^1 [(1-t^2)/(1-k^2t^2)]^{\frac{1}{2}}dt,$$

the complete elliptic integrals of the first and second kinds, respectively.

As a numerical example, assume that $\alpha=30°$, $R=5$ cm, $l=100$ cm, $g=980$ cm/sec^2. Then $k^2=0.460$, whence[2] $\mathbf{E}=1.36$ and $\mathbf{K}=1.84$, so that $T=0.73$ sec. This result is to be compared with the time of 0.90 sec for the non-unrolling tape descending the same incline.

I am indebted to Dr. J. M. Jauch for an interesting discussion of this problem.

[2] Jahnke and Emde, *Tables of functions* (Dover Publications, New York, 1943), p. 85.

Choosing Galvanometers for Wheatstone Bridges and Potentiometers

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In teaching a course in electrical measurements one is often faced with the problem of choosing adequate galvanometers for given Wheatstone bridge slide-wires and for potentiometers. Criterions for choosing reference resistance values, resistance ratios, galvanometer resistances, and voltages for Wheatstone bridges are given in some of the textbooks on electricity.[1] However, the writer has not been able to find in the ordinarily used literature a simple specific basis for determining the figure of merit[2] of a galvanometer that will allow a given bridge to attain its greatest sensitivity. A similar situation apparently exists in the choice of galvanometers that will permit maximum sensitivity with a given potentiometer and circuit.

The expressions derived herein have been found to be helpful and time-saving aids in the laboratory and in instruction; moreover, they can be understood and utilized by students.

[1] For example, see Page and Adams, *Principles of electricity* (Van Nostrand, 1931), pp. 175–180.

[2] Galvanometer sensivitity may be described by several terms, of which *current sensitivity* and *figure of merit* are most suitable here. Unfortunately, current sensitivity is defined in two different (reciprocal) ways in electricity textbooks. But textbooks agree in defining figure of merit, and so this term is used here.

Wheatstone Bridge

Consider a Wheatstone bridge of the slide-wire type (Fig. 1). Let X be the unknown resistance; R, the known reference resistance; a, the length of the portion of the slide-wire between the left-hand end and the sliding contact point; c, the total length of the slide-wire; V, the potential difference (assumed constant) applied to the bridge; k, the resistance per unit length of the slide-wire; G, the galvanometer, of resistance g. By applying Kirchhoff's laws to the circuit in the usual way, one finds that the current i through the galvanometer is

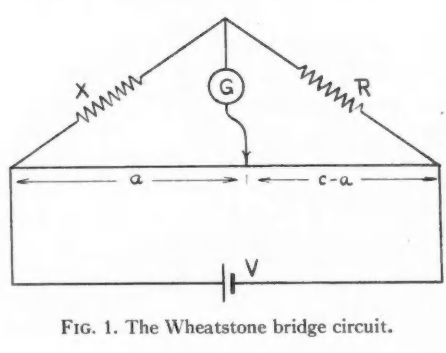

Fig. 1. The Wheatstone bridge circuit.

$$i=V\frac{X(c-a)-Ra}{(R+X)[gc+ka(c-a)]+RXc}. \quad (1)$$

We see that the condition for balance—$i=0$—is the well-known equation $X=Ra/(c-a)$.

To obtain an expression for the current through the galvanometer when the bridge is slightly unbalanced, the expression for the current i is differentiated partially with respect to a; this yields

$$\frac{\partial i}{\partial a}=-V(R+X)\frac{(R+X)[gc+ka(c-a)]+RXc+k(c-2a)[X(c-a)-Ra]}{\{(R+X)[gc+ka(c-a)]+RXc\}^2}. \quad (2)$$

Then this derivative is evaluated at the balance point by substituting for X its value $Ra/(c-a)$; thus,

$$\left(\frac{\partial i}{\partial a}\right)_{\text{bal}}=-\frac{V}{gc+ka(c-a)+Ra}. \quad (3)$$

The galvanometer current δi for the slightly unbalanced condition is given by $\delta a(\partial i/\partial a)_{\text{bal}}$, where δa, the smallest significant change in a, is determined by the constructional details of the slide-wire and scale and by the operator's ability to read the value of a; thus,

$$|\delta i|=\frac{V\delta a}{gc+ka(c-a)+Ra}. \quad (4)$$

The galvanometer employed should have a figure of merit F sufficiently small that a change of only δa in the position of the sliding contact will result in a significant deflection. The figure of merit is here considered to be the current required to produce a unit deflection on the galvanometer scale. Let θ be the smallest significant deflection on the scale. Then the galvanometer chosen should be such that the inequality

$$F\theta\leqslant\frac{V\delta a}{gc+ka(c-a)+Ra} \quad (5)$$

is satisfied.

Examination of inequality (5) indicates that the upper limit on the figure of merit decreases as V or δa is decreased, and as g, k, c or X (keeping $R\approx X$) is increased. The effect of increasing g is complicated by the fact that F and g are not independent. In fact, for a given kind of galvanometer,[1] F is approximately inversely proportional to $\sqrt{g}$. The behavior of $F\theta$ (θ is constant) as a function of g for a galvanometer is indicated in Fig. 2. The variation of the upper limiting value of $F\theta$ for an adequate galvanometer is also shown. A galvanometer which is possibly suitable must, of course, be such that $F\theta$ will be less than the upper limit on its value for some reasonable value of g. The graph of the two variables shows clearly that there is a minimum value of g for a galvanometer that can satisfy inequality (5). This minimum value of g can be obtained by tedious computations

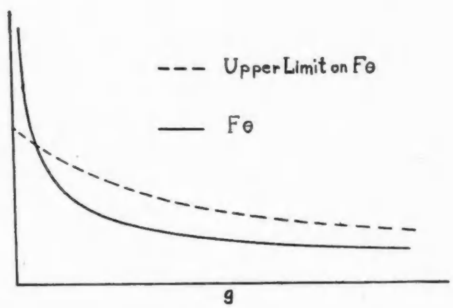

FIG. 2. $F\theta$ of a galvanometer and the upper limit on $F\theta$ for an adequate galvanometer *vs.* the resistance of the galvanometer.

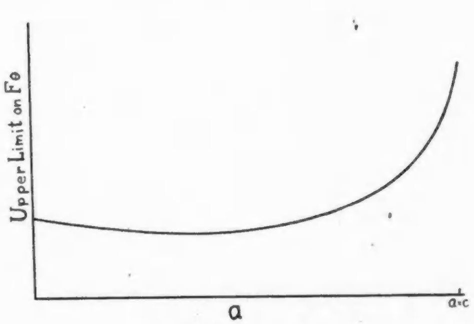

FIG. 3. Upper limit on $F\theta$ for an adequate galvanometer *vs.* the length a.

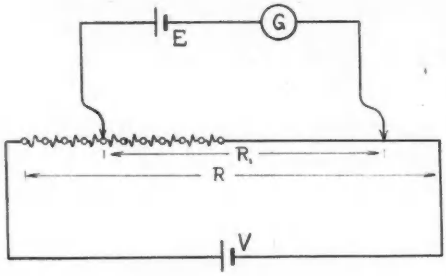

FIG. 4. The potentiometer circuit.

involving bridge and galvanometer characteristics. In practice, computation is not necessary; a laboratory usually has several possibly suitable galvanometers whose known characteristics are simply substituted in inequality (5) till one of the galvanometers is determined to be adequate.

The effect of a change in a (and, of necessity, also in R for a given value of X) on the upper limit of $F\theta$ is not so readily apparent. However, analysis of the right-hand member of inequality (5) shows that it varies with a about as indicated in Fig. 3. It can also be shown that the minimum exhibited there can never occur at a value of a greater than $\frac{1}{2}c$. Thus it may be concluded that if the galvanometer used is sufficiently sensitive at $\frac{1}{2}c$, it is then entirely adequate for all larger values of a.

As an example of the application of inequality (5), suppose that one wishes to measure a resistance of about 100 ohms and that a Leeds and Northrup Students' slide-wire is available. A dry cell ($V=1.5$ v) and a reference resistance of 100 ohms are used. For this slide-wire, $c=1000$ div, $\delta a=1$ div, and $k=0.3$ ohms/div; a is in this case observed to be about 500 div. Suppose there are available several galvanometers whose resistances are each about 50 ohms. Substitution of these values gives $F\theta \leqslant 8.6$ μamp. If then θ is considered to be 1 div on the galvanometer scale, a galvanometer should be used whose figure of merit is less than 8.6 μamp/div.

Potentiometers

Consider the simplified circuit for a null potentiometer shown in Fig. 4. Let E be the electromotive force generated by some device; R_1, the resistance between the two contact points; R, the total resistance of what is essentially (or actually) a slide-wire; and V, the potential difference (assumed constant) applied to the potentiometer; G is a galvanometer. Let the combined resistance in E and G be denoted by r. Application of Kirchhoff's laws to the circuit gives for the current i through the galvanometer,

$$i=\frac{ER-VR_1}{R_1(R-R_1)+rR}. \tag{6}$$

The condition for balance—$i=0$—is evidently represented by $E=R_1V/R$.

The procedure for obtaining an expression for the current through the galvanometer when the potentiometer is slightly unbalanced is the same as in the case of the Wheatstone bridge. The expression for the current i is differentiated partially with respect to R_1, and the derivative is evaluated for the balanced condition. This yields

$$\left(\frac{\partial i}{\partial R_1}\right)_{\text{bal}}=-\frac{V}{R_1(R-R_1)+rR}. \tag{7}$$

Then the galvanometer current δi for the slightly unbalanced condition is given by $\delta R_1(\partial i/\partial R_1)_{\text{bal}}$, where δR_1 is the smallest significant change in the value of R_1. As in the case of the Wheatstone bridge, the magnitude of δR_1 is a characteristic of the potentiometer and of the inherent error in reading R_1; we find that

$$|\delta i|=\frac{V\delta R_1}{R_1(R-R_1)+rR}. \tag{8}$$

The galvanometer used should have a figure of merit so small that a change δR_1 in R_1 will result in a significant deflection θ. Therefore, the galvanometer characteristics should satisfy the inequality

$$F\theta \leqslant \frac{V\delta R_1}{R_1(R-R_1)+rR}. \tag{9}$$

Inequality (9) has been developed in terms of resistance values rather than slide-wire lengths because the latter are not directly applicable for commonly used potentiometers. For the elementary slide-wire types, inequality (9) is applicable after resistances are expressed in terms of slide-wire lengths.

Since the approximate value of E is usually known before R_1 is computed, it is often easier to use a different inequality, equivalent to (9), namely,

$$F\theta \leqslant V\delta R_1/R\left[\frac{ER}{V}\left(1-\frac{E}{V}\right)+r\right]. \tag{10}$$

Inequality (10) indicates that low resistance potentiometers require less sensitive galvanometers since the upper limit on $F\theta$ decreases as R increases. However, for a given standardized potentiometer, only the values of E and r are subject to choice. It can easily be shown from inequality (10) that the upper limit on $F\theta$ has a minimum for $E=\frac{1}{2}V$. Thus a galvanometer adequate for measurement when $E=\frac{1}{2}V$ is sufficiently sensitive for measuring any value of E (less than V). Consideration of inequality (9) or (10) and the dependence of the figure of merit of a galvanometer on its resistance, in a manner exactly like that for the Wheatstone bridge, yields the same conclusion: the resistance of an adequate galvanometer must exceed a minimum value. Galvanometers whose resistances are less than this value cannot have a small enough figure of merit. It must be remembered that, with either the potentiometer or the Wheatstone bridge, the value of the critical damping resistance of the galvanometer should be less than the effective resistance of the circuit connected to the galvanometer, so that the latter can quickly respond to changes in balance. This often means that a series resistance must be added to the galvanometer circuit and it must be considered as part of r.

As an example, let inequality (9) be applied to an elementary slide-wire potentiometer, using typical values. Let $a=1000$ div; $c=2000$ div; $\delta a=2$ div; $k=0.025$ ohm/div; $r=100$ ohms; $V=2$ v; and $\theta=1$ div on the galvanometer scale. Also, $R_1=ka$; $R=kc$; $\delta R_1=k\delta a$. Then the figure of merit of a suitable galvanometer should be less than 17.8 μamp/div and, of course, its resistance should be such that r is 100 ohms.

For the Leeds and Northrup Students' potentiometer typical values are: $R_1=100$ ohms; $R=160$ ohms; $\delta R_1=0.054$ ohm; $r=100$ ohms; and $V=1.60$ v. Substitution in inequality (9) indicates that a suitable galvanometer should have a figure of merit less than 3.9 μamp/div.

An application to the Leeds and Northrup Type K-2 potentiometer might involve these values: $R_1=50$ ohms; $R=80.5$ ohms; $\delta R_1=0.002$ ohm; $r=100$ ohms; and $V=1.61$ v. Here the figure of merit should be less than 0.34 μamp/div.

The upper limits on the value of F just computed for the successively better types of potentiometers show well their increasing precision. An awareness of this progression in required sensitivity of adequate galvanometers is valuable to the student of electrical measurements.

There seems to be no definite basis for determining how much less the figure of merit of an adequate galvanometer should be than its upper limiting value. For Wheatstone bridges and potentiometers the author has arbitrarily taken this view: a galvanometer should not be used whose figure of merit is less than one-fifth the upper limiting value. To support this view, the experience of a student using a potentiometer is cited. He happened to use a galvanometer whose figure of merit was one-eighteenth the upper limiting value. It was evidently much too sensitive. A nonsignificant change in R_1 from the balance point gave a galvanometer deflection of several divisions. In fact, the smallest change in R_1 possible by human manipulation resulted in a galvanometer deflection of more than 1 div.

I CANNOT express the amazed awe, the crushed humility, with which I sometimes watch a locomotive take its breath at a railway station, and think what work there is in its bars and wheels, and what manner of men they must be who dig brown iron-stone out of the ground and forge it into that. What assemblage of accurate and mighty faculties in them, more than fleshly power over melting crag and coiling fire, fettered and finessed at last into the precision of watchmaking; Titanian hammer-strokes beating out these glittering cylinders and timely respondent valves, and fine ribbed rods, which touch each other as a serpent writhes in noiseless gliding and omnipotence of grasp, an infinite complex anatomy of active steel. What would the men who beat this out, who touched it with its polished calm of power, who set it to its appointed task and triumphantly saw it fulfill the task to the utmost of their will, feel or think about this weak hand of mine timidly leading a little stain of water color which I cannot manage into the imperfect shadow of something else . . . what, I repeat would these iron-dominant genii think of me, and what ought I to think of them?"—JOHN RUSKIN.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

25. Richard Trevithick and the First Railway Locomotive

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"It has been claimed that the steam locomotive, in conjunction with the railway, has done more to promote the progress of the human race than any other single product of man's ingenuity."[1]

The originator of the steam locomotive was, without question, Richard Trevithick (1771–1833), who pioneered the high pressure noncondensing steam engine. (Plate 1.) On Christmas Eve, 1801, on Beacon Hill at Camborne in Cornwall, a road locomotive, designed and built by Trevithick, carried the first load of passengers ever conveyed by steam; and, in 1803, another steam vehicle made by him was run in the streets of London without, strange as it now seems, attracting enough attention to lead to a single press notice. A modification of this engine was successfully used in 1804 to haul trucks on the tramway running from the Penydarran Iron Works, near Merthyr Tydvil, Wales, to Abercynon, a distance of nine miles. This was the first railway locomotive. Unfortunately its exact construction is uncertain and no authentic illustration of it exists. It is known, however, that it weighed about 5 tons and had a single cylinder 8.25 in. in diameter with a 54-in. stroke. It also discharged the exhaust steam into the chimney. When first tried on February 21, 1804, a date forever memorable in the history of the locomotive, it hauled a load of about 20 tons at a speed of 5 mi/hr. The performance was described by Trevithick in a letter to Davies Giddy, dated "Penydarran, 1804, Feb. 22nd," which reads as follows:[2]

Yesterday we proceeded on our journey with the engine; we carry'd ten tons of Iron, five waggons, and 70 Men riding on them the whole of the journey. Its above 9 miles which we perform'd in 4 hours & 5 Mints, but we had to cut down som trees and remove some Large rocks out of the road. The engine, while working, went nearly 5 miles pr hour; there was no water put into the boiler from the time we started untill we arriv'd at our journey's end. The coal consumed was 2 Hund[d]. On our return home, abt 4 miles from the shipping place of the Iron, one of the small bolts that fastened the axel to the boiler broak, and let all the water out of the boiler, which prevented the engine returning untill this evening. The Gentleman that bet five Hund[d]. Guineas against it, rid the whole of the journey with us and is satisfyde that he have lost the bet. We shall continue to work on the road, and shall take forty tons the next journey. The publick untill now call'd mee a schemeing fellow but now their tone is much alter'd.

Plate 1. Richard Trevithick (1771–1833). [From the oil painting made in 1816 by John Linnell, which is now in the Science Museum, London.]

[1] Handbook of the Science Museum, *Land transport. III· Railway locomotives and rolling stock.* Part I "Historical review" (H. M. Stationery Office, London, 1931). The writer is greatly indebted to the Science Museum and its excellent handbooks.

[2] From the Trevithick—Giddy correspondence preserved by the Enys family at Enys and now in the possession of the Royal Institution of Cornwall at Truro, Cornwall.

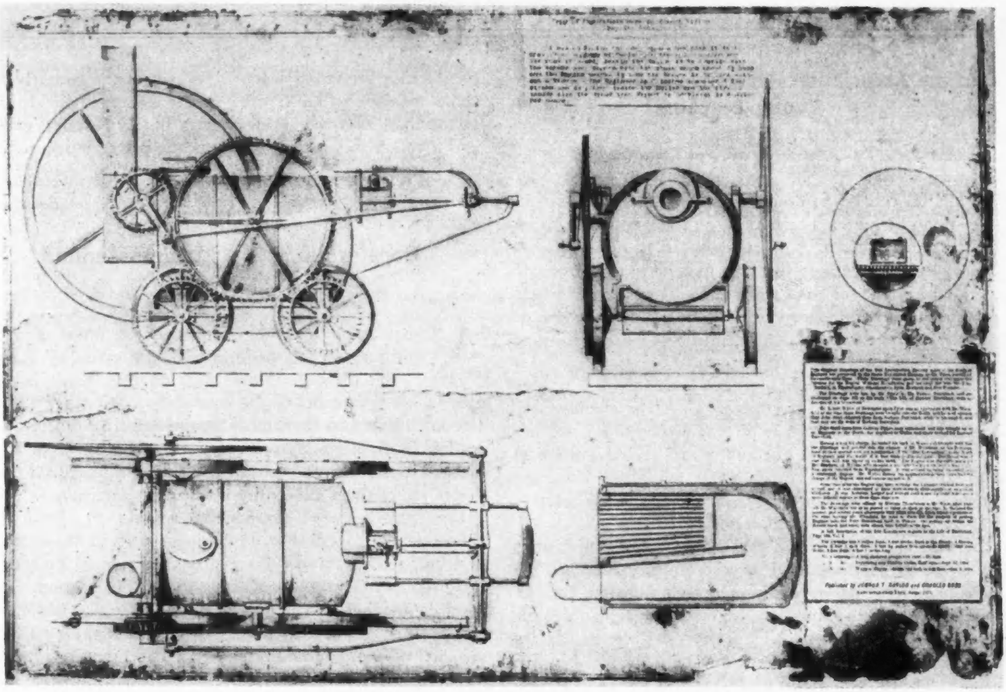

Plate 2. TREVITHICK'S NEWCASTLE LOCOMOTIVE, 1805. [From the original drawing in the Science Museum, London.]

A similar engine was supplied to the Wylam colliery at Newcastle in 1805, but apparently it was not used, possibly because its weight was too great for the wooden rails upon which it was to run. Drawings of this locomotive have, however, been preserved and are now in the Science Museum in London. They are reproduced in Plate 2. This locomotive is believed to be almost identical with the Penydarran one and is the lineal ancestor of the Wylam[3] and early Stephenson locomotives.

Several accounts of TREVITHICK's romantic career are available. *The Life of Richard Trevithick, with an Account of his Inventions*, written by his son, FRANCIS TREVITHICK, (London, 1872, 2 vol.) is the source of nearly all that is known about him, but it is uncritical, tedius and repetitious. A more critical, readable and up-to-date life has been provided by W. H. DICKINSON and ARTHUR TITLEY in their memorial volume, *Richard Trevithick, the Engineer and the Man* (Cambridge, 1934). This excellent work contains a very complete four-page bibliography. A still more recent short account will be found in *Great Engineers*, by C. MATSCHOSS (London, 1939, pp. 154–171).

[3] See the ninth reproduction in this series, *Am. J. Physics* **8**, 46 (1940).

SCIENCE is the great instrument of social change, all the greater because its object is not change but knowledge, and its silent appropriation of this dominant function, amid the din of political and religious strife, is the most vital of all the revolutions which have marked the development of modern civilization.—A. J. BALFOUR, *Decadence* (1908).

NOTES AND DISCUSSION

Can the Association Recognize and Encourage Young Teachers?

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THERE can be no doubt that the American Association of Physics Teachers is, and will continue to be, a flourishing society. Nevertheless, it is disturbing to observe at the meetings that the members do not constitute a true cross section of American college physics teachers. They are middle-aged or elderly; young teachers do not find in the Association a natural professional meeting place.

That the Association should have been founded by men past their youth was inevitable. That the membership should continue to be mostly of that description is neither necessary nor desirable. Far too many persons, some of them known to the author, delay affiliation with the Association until mellowed by middle age; they tend to let it be the last professional organization tried.

One reason for this is the fact that a *young* man cannot now hope to be recognized as a promising teacher; the mechanism for such recognition does not exist. On the other hand, a young man who shows promise in research may be elected to Sigma Xi; in any case, the professors at his graduate school will have formed a rather accurate estimate of his potentialities for a research career, although they will be much more vague if asked to predict his suitability for a purely teaching position.

The Association does recognize and encourage achievement in teaching, but not by young teachers. The Oersted Medal, from its very nature, is awarded at or near the end of a long career. Election to office in the Association, committee membership, even invited papers at the meetings, are available for persons of middle age rather than for young men. Few young teachers, no matter how great their talents, can hope to compete upon equal terms with men of substantial length of service in any of these respects.

It has been freely predicted that in the postwar years there will be many more industrial positions open for men trained in physics than were previously available, so that there will be a scarcity of young men anxious to enter the profession of teaching. This scarcity, and means of reducing it, are the proper concern of the Association. With the usual disparity between academic and industrial salaries, it is useless to expect that many persons will leave industry, after several years, to enter upon a teaching career at middle age. The supply of teachers must be replenished each year by the entry of *young* men directly into teaching, and their retention therein. The problem is to make a teaching career attractive to a young man; part of the problem would be solved by the prospect that a good job of teaching will be recognized as such, and identified by suitable ***kudos***.

Can the American Association of Physics Teachers manage to provide recognition for young men who are considering becoming teachers, or who have recently become teachers? Can it offer any encouragement to the young man who wavers between a purely research career and a career in which teaching is the primary occupation and interest?

Doppler Effect in a Moving Medium

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AN expression for the Doppler effect under general conditions of the medium has been given by R. W. Young.[1] The present note arrives at this result by an alternative treatment of the sound ray. Attention is particularly drawn to the change in the velocity of sound resulting from the transverse motion of the medium. This method describes the physical principles involved and traces the changes caused by the various factors.

Let there be wind components w_1 and w_2 along and perpendicular to AB (Fig. 1). Suppose that, in the absence of wind, sound travels along any direction χ, and that AD_1 represents c, the ordinary velocity of sound. The wind component w_1 carries the point D_1 to D_2, while the wind component w_2 carries D_2 to D_3; thus the resultant velocity of sound is AD_3, or c_1, in the direction θ with respect to AB. Taking projections perpendicular and parallel to AB, we get

$$c \cos \chi = c_1 \sin \theta + w_2,$$

and

$$c \sin \chi = c_1 \cos \theta - w_1.$$

Hence, on eliminating χ,

$$c_1 = (c^2 - W^2 \cos^2 \psi)^{\frac{1}{2}} + W \sin \psi, \tag{1}$$

where $\psi = \phi - \theta$, $\tan \phi = w_1/w_2$ and $W^2 = w_1^2 + w_2^2$. If W_1 and W_2 represent the wind components parallel and perpendicular to AD_3, then Eq. (1) can be written in the form

$$c_1 = (c^2 - W_2^2)^{\frac{1}{2}} + W_1. \tag{2}$$

Let us now evaluate the Doppler effect when the source is traveling with velocity u along AB and the observer is moving along PB (Fig. 2) with velocity u', PB being

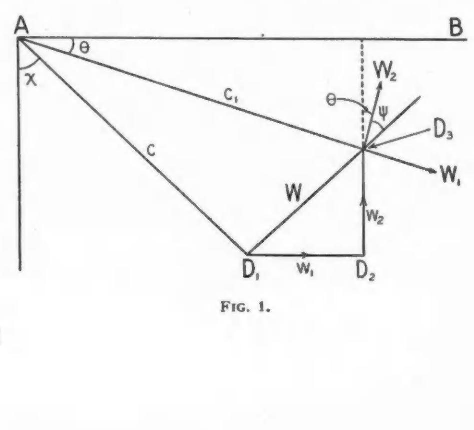

FIG. 1.

perpendicular to AB. Let A represent the position of the source at any instant. Rays starting from A reach the observer at P after an interval t_1 given by $t_1=AP/c_1$. In a small interval Δt the source moves to A', so that $AA'=u\Delta t$; rays starting from A' reach the observer at P' after a time t_2 given by $t_2=A'P'/c_1'$, where c_1' is the resultant velocity along $A'P'$. If dt represents the interval between the times of arrival of the rays from A and A' at the observer, then

$$dt=\Delta t+\frac{A'P'}{c_1'}-\frac{AP}{c_1}. \tag{3}$$

Since the number of waves emitted by the source in time Δt is $\nu\Delta t$, and the same number is received by the observer in time dt, we have

$$\nu\Delta t=\nu' dt, \tag{4}$$

where ν is the frequency of the source and ν' the frequency as heard by the observer. From Fig. 2 we find

$$r\alpha=u\Delta t\sin\theta,\quad r\beta=u'dt\cos\theta, \tag{5}$$

$$AP=r+u\Delta t\cos\theta,\quad A'P'=r-u'dt\sin(\theta+\alpha), \tag{6}$$

while

$$\begin{aligned}c_1'&=[c^2-W^2\cos^2(\psi-\Delta\psi)]^{\frac{1}{2}}+W\sin(\psi-\Delta\psi)\\&=(c^2-W^2\cos^2\psi)^{\frac{1}{2}}-\frac{W^2\cos\psi\sin\psi\Delta\psi}{(c^2-W^2\cos^2\psi)^{\frac{1}{2}}}\\&\qquad+W\sin\psi-W\cos\psi\Delta\psi\\&=c_1-W\cos\psi\Delta\psi-\frac{W^2\cos\psi\sin\psi\Delta\psi}{(c^2-W^2\cos^2\psi)^{\frac{1}{2}}}\\&=c_1-\frac{c_1W\cos\psi\Delta\psi}{c_1-W\sin\psi}\\&=c_1\left\{1-\frac{W\cos\psi\Delta\psi Q}{c_1}\right\},\end{aligned} \tag{7}$$

where $\Delta\psi=\alpha-\beta$, and $Q=(1+W/c_1\sin\psi)$, $W\ll c$. Inserting in Eq. (3) the values of AP, $A'P'$ and c_1' from Eqs. (6) and (7), neglecting small quantities, and making use of Eqs. (4) and (5), we get

$$\frac{\Delta t}{dt}=\frac{\nu'}{\nu}=\frac{1+\dfrac{u'}{c_1}\sin\theta+\dfrac{u'WQ\cos\theta\cos\psi}{c_1^2}}{1-\dfrac{u}{c_1}\cos\theta+\dfrac{uWQ\sin\theta\cos\psi}{c_1^2}}.$$

If U_0, V_0; U_1, V_1; W_1, W_2 represent the velocity components of the source, the observer and the wind, respectively, parallel and perpendicular to AP, we have

$$\frac{\nu'}{\nu}=\frac{1-\dfrac{U_1}{c_1}+\dfrac{W_2V_1}{c_1^2}+\dfrac{W_1W_2V_1}{c_1^3}}{1-\dfrac{U_0}{c_1}+\dfrac{W_2V_0}{c_1^2}+\dfrac{W_1W_2V_0}{c_1^3}}.$$

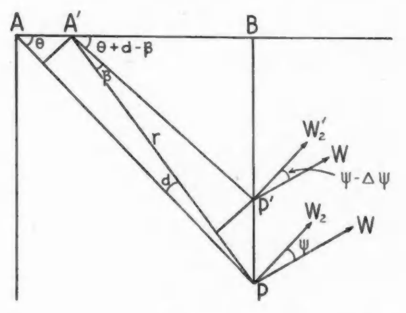

FIG. 2.

The first two terms in the numerator and denominator of the right-hand member show the ordinary Doppler effect due to motion of source and observer along the same line. The third and the fourth terms show the modification due to transverse motion of source and observer and the wind. If V_0 and V_1 are zero, then the third and fourth terms disappear. In that case the modification is introduced in the second term by transverse wind affecting the resultant velocity of sound. Hence, even if there is no transverse motion of the observer or the source, the transverse wind component modifies the ordinary Doppler effect. This is the point of view that Barton[2] had in mind. If the transverse component of wind is greater than c, then c_1 becomes imaginary. This means that no sound can reach an observer.

[1] Young, *J. Acous. Soc. Am.* **6**, 112 (1934).
[2] E. H. Barton, *A textbook on sound* (Macmillan, 1908), p. 98.

Preparation of Graphs for Physical Papers

THESE suggested practices are intended for the guidance of authors and draftsmen in preparing graphs for reproduction in technical publications. No attempt has been made to provide a comprehensive set of suggestions, but merely to emphasize certain practices that often are disregarded. Although the discussion will be confined to the simplest types of graph, many of the suggestions are applicable to line drawings in general. Most of the recommendations are in accord with those recently made by a committee of the American Standards Association.[1]

Curves.—Not more than three or four curves ordinarily should be shown on the same graph, although more may be included in the case of a family of well-separated curves. If one curve is especially important, a solid line should be used for it, and dashed, dotted or lighter solid lines for the others.

No curve or coordinate ruling of the graph should run through any lettering or through outlined circles, triangles, and so forth, that are used to indicate plotted points. (Compare Figs. 1 and 2.)

Coordinate rulings.—Coordinate rulings should be limited in number to those needed to guide the eye in making a reading to the desired degree of approximation. Short scale markers, or "ticks," may be inserted between rulings if this is desirable (Fig. 2). The rulings should be made light enough not to distract attention from the curves being presented.

Orientation and location of lettering.—All lettering should be placed so as to be easily read from the bottom and from

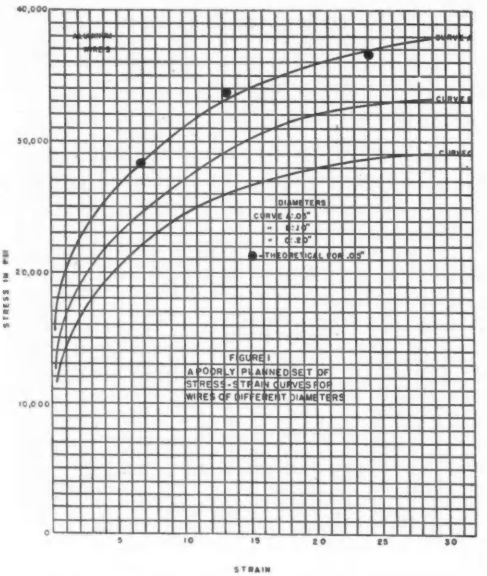

FIG. 1. In this drawing some ten principles of good graph making are violated. Figure 2 contains precisely the same information, but is easier to interpret and requires less space.

the right-hand side of the graph; that is, the lettering should face either the bottom or the right-hand side of the drawing.

A graph should be free of all lines and lettering that are not essential for clear understanding. As far as practicable, explanatory comments, supplementary data or formulas should be placed, not on the face of the graph, but in the figure legend or in the text. The main exception to this rule is in the case where there are several curves on the same graph that need separate identification; if practicable, they should be identified by brief labels placed close to the curve (horizontally or along the curve) rather than by single letters or numbers requiring a key.

If it seems necessary to place supplementary information on the drawing proper, the lettering should be kept within the vertical and the horizontal limits of the curves or other essential features of the drawing (Fig. 2). If the labels for the curves are arranged as in Fig. 1, for example, the space occupied by the drawing will be needlessly large, or else the drawing will have to be reduced in reproduction, often to the point where the lettering or other details are illegible.

Scale captions.—The scale captions should be placed outside the grid area, normally at the bottom toward the right for the horizontal scale and at the left-hand side toward the top for the vertical scale.

The scale caption should contain, in the order named, (i) the standard name of the physical variable plotted, (ii) its symbol, if one is used in the text, and (iii) the abbreviation for the unit of measure. The standard abbreviation for the unit should be employed, and this abbreviation should be placed in parentheses following the name of the quantity; thus,

PRESSURE p (LB/IN.^{2})
RESISTANCE R (OHM)
VELOCITY (10^3 FT/SEC).

Avoid using such captions as "Pressure in lb/in.^{2}," "Pressure in lb per sq in." and "Pressure in pounds per square inch."

The technical terms, symbols and abbreviations appearing on a drawing should be in accord with those used in the text of the article. If there is uncertainty as to the best practice in a particular case, the lettering should be done initially with pencil and inked in only after the paper has been reviewed and edited.

Scales and scale values.—The horizontal and vertical scales for a graph should be chosen with care, so as to give a correct impression of the relationship plotted, for the choice of scales has a controlling influence on the apparent rate of change of the dependent variable.

Except in cases where a visual comparison of plotted magnitudes is important, the bottom (abscissa) and extreme left-hand (ordinate) coordinate lines need not necessarily represent the zero values of the variables plotted. If this suggestion is followed where feasible, space can be saved and the presentation often rendered more effective.

The numerals representing the scale values should be placed outside the grid area. If the scale values are smaller than unity and are expressed in decimal form, a cipher should always precede the decimal point; thus, 0.20, not .20.

The use of many ciphers in scale numbers should be avoided, and the best way to do this is to re-express the quantity plotted in terms of a larger unit of measurement. For example, suppose that originally the scale numbers are 15 000, 20 000, 25 000··· and that the scale cap-

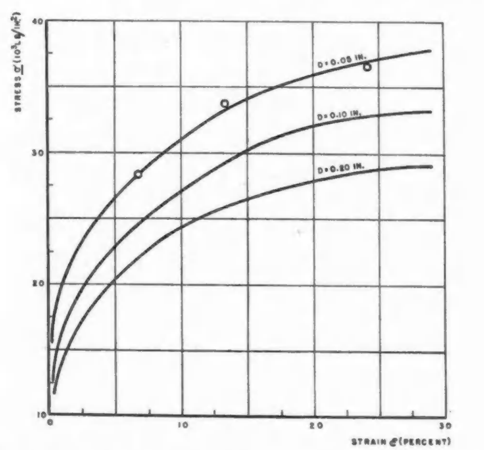

FIG. 2. A well-planned set of experimentally determined stress-strain curves for aluminum wires of different diameters D. The three plotted points O represent values for the 0.05-in. wire predicted by Meeting's theory.

tion is "Pressure (lb/in.²)"; these scale numbers can be changed to 15, 20, 25, ···, provided the scale caption is made

PRESSURE (10^3 LB/IN.²).

If, in this example, the data are correct to three significant figures and it is desirable to indicate this fact, then the scale values should be 1.50, 2.00, 2.50, ···, and the scale caption,

PRESSURE (10^4 LB/IN.²).

Never use captions of the following types: "Velocity in thousands of feet per second," "Velocity$\times 10^3$ in ft/sec" and "Velocity (ft/sec$\times 10^3$)." The first requires too much space and lettering; the remaining two are ambiguous, since they do not indicate clearly whether the scale numbers *have been* or *are to be* multiplied by 10^3.

Figure legend.—A brief legend should be provided for each graph, but it should not be made a part of the drawing. All legends are set in type, and hence should be typewritten double-spaced, in a list on the final page of the manuscript. On the margin of each drawing there should appear in pencil the number of the figure, the name of the author and the title of the article.

Ink and paper.—Black drawing ink should be used. Many writing inks do not reproduce well. Drawings should be made on tracing cloth, tracing paper, or a fairly heavy white paper having a dull surface that will stand erasure without roughening. Bristol board provides a satisfactory drawing surface, but is easily damaged in transit. Ruled coordinate paper may be used, provided it is printed in light blue; the important coordinate lines and scale markers that are intended to appear in the reproduction must be ruled in ink.

Size of drawing.—A good size for a drawing is double that desired for the printed figure; *all lettering and line thicknesses should be increased accordingly.* Thus, a drawing that is to be reproduced column-width in the AMERICAN JOURNAL OF PHYSICS should be made about 14 cm wide over-all. A simple drawing containing little detail may often be so planned that the printed figure can be made less than column-width.—D. R.

[1] *Engineering and scientific graphs for publications*, ASA publication Z15.3 (American Standards Association, 70 East 45th St., New York, N. Y.), price 75 cts. This useful pamphlet contains 71 specific recommendations covering all phases of the subject.

Check List of Periodical Literature

Black and white. N. R. Hall, *Sch. Sci. Rev.* **25**, 49–57 (1943). Some notes on vision without special reference to color.

Experiments with simple valve voltmeters. L. S. Joyce, *Sch. Sci. Rev.* **25**, 57–62 (1943). Simple diode and triode voltmeter circuits.

Experiments on the physics of transpiration. L. G. G. Warne, *Sch. Sci. Rev.* **25**, 66–70 (1943). Simple demonstrations of the effect of incipient drying, diffusion shells, evaporation through slits of various sizes, through a multiperforate septum, and from open cylinders of different depths.

Optical aids in education. H. E. Dance, *Sch. Sci. Rev.* **25**, 172–179 (1944). Uses of sound film, lanternslides, film strips, microfilm.

Resonant circuits. L. S. Joyce, *Sch. Sci. Rev.* **25**, 185–195 (1944). Series and parallel resonant circuits with particular reference to radio.

Causation and modern physics. W. E. Matthews, *Sch. Sci. Rev.* **25**, 305–311 (1944).

Mechanics of bicycle stability. G. T. P. Tarrant, *Sch. Sci. Rev.* **25**, 321–332 (1944). A mathematical discussion, including the problem of riding without holding on to the handlebars.

Functional English. R. O. Kapp *Sch. Sci. Rev.* **26**, 2–7 (1944). A plea for better instruction in writing technical English, particularly for engineers.

Coated lenses. A. Harvey, *Sch. Sci. Rev.* **26**, 36–44 (1944).

Laboratory experiments on the wave theory of light. S. Weikersheimer, *Sch. Sci. Rev.* **26**, 44–48 (1944). Simple experiments on interference and diffraction.

On pattern and design. R. C. Evans, *Sch. Sci. Rev.* **26**, 295–307 (1945). A simple account of space-group theory, chiefly in two dimensions, and its relations to crystallography.

Home-constructed spectroscope. W. M. Baxter, *Sch. Sci. Rev.* **26**, 311–314 (1945). Directions for making a simple grating spectroscope, using a special kind of cellophane for a grating.

A simple air compressor. G. H. Wilson, *Sch. Sci. Rev.* **26**, 307–311. A one-cylinder, two-stroke motorcycle engine is used.

Flagellar movement. A. G. Lowndes, *Sch. Sci. Rev.* **26**, 319–332 (1945). High-speed motion pictures have shown the method by which such organisms as Euglena propel themselves by flagellar movement. The mechanics of the motion are discussed, and it is pointed out that several published accounts are in error.

Symbols and definitions in (i) electricity and magnetism, (ii) illumination and photometry. *Sch. Sci. Rev.* **26**, 362–368 (1945). Recommendations of the Committee for coordination and guidance in physics teaching of the Physical Society and the Science Masters' Association.

Proceedings of the American Association of Physics Teachers

The New York Meeting, January 24–26, 1946

THE fifteenth annual meeting of the American Association of Physics Teachers was held at Columbia University, New York City, on January 24 to 26, 1946.

A joint dinner with the American Physical Society was held at the Men's Faculty Club of Columbia University on Friday evening, January 25.

Invited Papers

Joint Symposium with the American Physical Society: Nuclear Energy

Scientific aspects. H. A. BETHE, *Cornell University.*

Social implications. A. H. COMPTON, *Washington University.*

International implications. JAMES T. SHOTWELL, *Carnegie Peace Foundation.*

Joint Session with the American Physical Society

Technological research in the university. P. E. KLOPSTEG, *Northwestern University Technological Institute; fourth Richtmyer Memorial Lecture of the American Association of Physics Teachers.*

The pitch, loudness and quality of musical tones. HARVEY FLETCHER, *Bell Telephone Laboratories; retiring president of the American Physical Society.*

Presentation of the Oersted Medal to Ray Lee Edwards. L. W. TAYLOR, *Oberlin College,* AND R. C. GIBBS, *Cornell University.*

Symposium on Physics and the Nation

Physicists in national affairs. M. H. TRYTTEN, *Office of Scientific Personnel.*

Federal support of science. E. U. CONDON, *The National Bureau of Standards.*

Addresses

Physics and the returning veteran. T. H. OSGOOD, *Michigan State College.*

Demonstration experiments. E. M. ROGERS, *Princeton University.*

Contributed Papers, with Abstracts

Two sessions were devoted to the following contributed papers.

1. Demonstration using a large low speed gyroscope. HAROLD K. SCHILLING, *The Pennsylvania State College.*—The complete paper appears elsewhere in this issue.

2. Demonstrations of the use of microwaves in teaching physical optics. C. L. ANDREWS, *New York State College for Teachers.*—In teaching the nature of electromagnetic waves, it is most effective to begin with waves that can be measured with an ordinary meter stick. Waves of frequency 2500 Mc/sec ($\lambda = 12$ cm) require apparatus of convenient size for laboratory and lecture experiments. The basic apparatus consists of a transmitter and intensity meter, each the size of a man's hand. The transmitter is an oscillator with two coupled resonant cavities employing a General Electric 2C43 disk-seal triode as an integral part of the cavities. The intensity meter is a crystal detector and microammeter. Demonstrations are made with microwaves of Young's experiment with interference from two secondary sources, Lloyd's mirror, standing waves, interference in thin films, diffraction by circular apertures, and polarization.

3. Effects incident to change of force. ERNA M. J. HERREY, *Queens College.*—An oscillator may be set into vibration with its natural frequency by an aperiodically changing force, for instance, by a force increasing from zero to a maximum value. The amplitude of the vibration is large if the time in which the force reaches its final value is short compared with the natural period of the oscillator. This phenomenon is observed in acoustics when sudden removal of a stopper from a bottle produces a loud report whose pitch is determined by the resonant frequency of the system. The analogous electrical effect is used in the transient method of testing communication circuits. Similar phenomena are now assuming significance in connection with modern high speed vehicles. The accelerations of such vehicles are high, yet not really constant, as is assumed in most calculations. This inevitably results in sudden changes of force. Particularly interesting are such changes if produced by vehicles entering or leaving curves, when the radial acceleration changes from zero to v^2/R and back to zero. The effects upon passenger and vehicle are important not only for highway but for vehicle design, especially for the design of transition spirals between straight and curved sections. Phenomena incident to change of force are of physical interest, and part of every student's daily experience; thus classroom discussion seems advisable.

4. Structure of cubic crystals as revealed by x-rays. S. S. SIDHU, *University of Pittsburgh.* (Introduced by A. G. Worthing.)—X-ray diffraction studies reveal cubic crystals with three types of unit cell: ***simple cube, body-centered cube*** and ***face-centered cube.*** The unit cell of a simple cube contains only one atom with coordinates (u, v, w) at (0, 0, 0); that of a body-centered cube, of two atoms at (0, 0, 0), ($\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$); and that of a face-centered cube, of four atoms at (0, 0, 0), ($\frac{1}{2}$, $\frac{1}{2}$, 0), ($\frac{1}{2}$, 0, $\frac{1}{2}$), (0, $\frac{1}{2}$, $\frac{1}{2}$). Interplanar

distance d between two parallel planes in the unit cell of any cubic crystal is equal to $a_0/(h^2+k^2+l^2)^{\frac{1}{2}}$, where a_0 is the length of the side of the unit cell and h, k, l are Miller indices of planes. The intensity of reflection I from h, k, l-planes is given by

$$I\alpha(F_{hkl})^2=[\Sigma f \cos 2\pi(hu+kv+lw)]^2 + [\Sigma f \sin 2\pi(hu+kv+lw)]^2,$$

where f is the atomic scattering factor, and F_{hkl} is the structure factor. Thus, in the unit cell of a simple cube the Miller indices h, k, l of planes that give reflections can have all values; in the body-centered cube, $h+k+l$ must be even; and in the face-centered cube, h, k, l are either all odd or all even. Structures of cubic crystals as described in some of the widely used textbooks of chemistry and physics are untenable according to the foregoing rules.

5. General equations in the field of electricity and magnetism that are not dependent on the system of units used in making computations. A. G. WORTHING, *University of Pittsburgh.*—In electromagnetism, the well-known equations $\oint \mathbf{H}\cdot d\mathbf{s}=4\pi nI$ of the emu system, $\oint \mathbf{H}\cdot d\mathbf{s}=(4\pi/c)nI$ of the Gaussian system, $\oint \mathbf{H}\cdot d\mathbf{s}=nI$ of the mks system, and $\oint \mathbf{H}\cdot d\mathbf{s}=(4\pi/10)nI$ of the hybrid, emu-mks system are used to describe the same phenomenon. Each equation is said to be correct only for certain systems of units, with each symbol standing for a number of units of measurement. The $\mathbf{H}$, $d\mathbf{s}$, $\cdots$ are said to represent a number of oersteds, a number of centimeters, $\cdots$. Since the same phenomenon is described by all four equations, it would seem that a single equation with symbols representing physical quantities should suffice. To replace the four equations just given, the single equation $\oint \mathbf{H}\cdot d\mathbf{s}=knI$ will suffice. Except for k, which is a transformation factor for converting units, the symbols now represent physical quantities. For the emu, the Gaussian, the mks, and the hybrid emu-mks systems, k in turn becomes 4π oersted cm/abamp turn, $(4\pi/C)$ oersted cm/statamp turn, 1 (amp turn/m)m/amp turn, and $(4\pi/10)$ oersted cm/amp turn. All of these expressions for k may be obtained by substitution as soon as one is known. As for all transformation factors, their physical equivalent is the numeric 1. Other basic relations in the field may be treated similarly.

6. The growth of the concept of mass-energy equivalence: a historical interpretation. RICHARD M. SUTTON, *Haverford College.*—A critical examination is made of the prerelativity factors which, at the turn of the century, contributed to the currency of certain ideas of electromagnetic mass and of the variability of inertial mass with velocity. The gathering body of experimental knowledge which has since accompanied the rapid growth of the concept of mass-energy equivalence into a principle of transcendent importance is reviewed. The variety of approaches to the remarkable concept is emphasized.

7. Procedures for the placement of transfer students with advanced standing in physics. BERNARD B. WATSON, *University of Pennsylvania.*—In connection with the Naval Academic Refresher Unit (V-7), procedures were worked out for selecting those men presenting college credit in general physics who are ready for courses requiring a knowledge of basic physical principles and those men who need either a short refresher course or a complete repetition of the general course before they can profitably undertake more advanced work. These procedures should also be applicable to veterans returning to college with advanced standing in physics. Standings on the Cooperative Physics Tests for College Students—mechanics, heat, sound, light and electricity—were used as the basis for selection. Men ranking in the 20th percentile or above in each of the five tests were found to require no refresher work. Those ranking in the 10th percentile or above in both mechanics and electricity, or in one of these and in both heat and light, could be brought to a satisfactory level of competence by a refresher course of eight weeks duration. Those failing to meet the standards for either of the afore-mentioned groups should repeat the general course.

8. A demonstration of the meaning of the Fourier components, AGNES TOWNSEND, *Barnard College* AND CHARLES F. WIEBUSCH.—The physics student who meets a Fourier analysis of a complex wave for the first time sees it as a mathematical device with no physical significance. An experiment has been designed to show that the Fourier harmonics exist and can be measured. The apparatus consists of a mass-loaded reed driven by a spring that rests against a rotating disk. This disk is cut out in the shape of the wave to be analyzed and can be rotated with various measured frequencies by means of a speed-changing mechanism attached to a motor. The amplitude of oscillation of the reed is measured with a light beam reflected from a mirror fixed on the reed. These amplitudes at the frequency of the fundamental and the harmonics give the Fourier coefficients at these frequencies. Coefficients determined in this way for some standard wave forms show good agreement with the calculated values.

9. The object of laboratory experiments in physics for liberal arts students. ERIC M. ROGERS, *Princeton University.*—A plea is made to reconsider the laboratory course with this question in mind: "What will contribute the greatest value to what the student will recall five or ten years later?" Facts and technics learned in the laboratory may contribute much less than will (a) understanding of physical concepts and of the experimental nature of physics and (b) a sense of delight in or respect for or enjoyment of physics, gained from laboratory work. We can encourage (a) and (b) by eliminating some routine measurements (specific heats) and technic practice (verniers), and adding some qualitative experiments (say, on radiation), some semi-original ones (say, with pendulums) and some complicated ones with romantic appeal (e/m for electrons). But the chief change needed is in the *student's* idea of the object of the experiment. He often thinks privately it is to please the teacher, to "get the answer by the book," or to "verify a law." He should have a less formal aim, such as "To experiment with. . . ," "To make tests of. . . ," "To see how . . . is measured"; and should

at almost all costs enjoy the experiments. Without enjoyment, laboratory work may in the long run do more harm than good.

10. Atoms in cartoons. ROBERT S. SHAW, *College of the City of New York.*—A sequel to an earlier paper [*Am. J. Physics* **11**, 47 (1943)].

11. Lecture demonstration of nodal patterns. E. R. PINKSTON, *United States Naval Academy.*—The vibration modes of plates are most readily demonstrated by the sand patterns discovered by Chladni.[1] The patterns can be made visible to a large audience by means of a mirror placed behind the plates and inclined somewhat forward.[2] If the plates are strongly illuminated, the sand particles are easily visible and sparkle as they move from one position to another. Vibrations of sufficient amplitude and a considerable range of frequencies can be obtained by using magnetostriction tubes driven by the amplified output of an oscillator. The plates may be supported upon some soft material and driven at the center, or they may be clamped at the center and driven by applying the magnetostriction tube at selected points. Nodal patterns obtained in this manner agree closely with those calculated from the plate constants by Ritz's method.[3] Some advantages of this method of setting up the vibrations over the old method of bowing are: (i) if the plate rests on felt and is driven by a light tube attached at the center, it vibrates very nearly as a completely free plate; (ii) if the plate is clamped at the center and driven at selected points, it is not necessary to furnish support along anticipated nodal lines and thus cause the mode of vibration to deviate from the natural node. The different patterns are easily and quickly obtained simply by varying the oscillator frequency.

[1] *Entdeckungen über die Theorie des Klanges* (1787).
[2] Sutton (Ed.), *Demonstration experiments in physics* (1938), pp. 187–188.
[3] *Ann. Physik* [4] **28**, 737 (1909).

12. Temperature coefficients of resistance, positive and negative. W. B. PIETENPOL, *University of Colorado.*—Inexpensive light bulbs may be used for an effective demonstration of the variation of electric resistance with change in temperature, including both negative and positive temperature coefficients of resistance. No galvanometer or external source of applied heat, such as a gas flame, is needed. A 110-v carbon lamp and a flashlight bulb of selected current capacity are connected in series with a switch to the 110-v circuit. When the switch is closed the small lamp lights up slowly, showing that the resistance of the carbon lamp filament decreases with increase in temperature. The demonstration of positive temperature coefficient is even more effective. A 60-w tungsten lamp may be used with a different indicator lamp. When the switch is closed the small lamp flashes brilliantly and then goes out. Momentary opening of the switch while the tungsten lamp filament remains hot results in no illumination of the small lamp. But, when the 110-v tungsten lamp filament is allowed to cool, the initial flash of the indicator lamp is repeated. The experiment may be performed with either alternating or direct current.

13. Device for showing object and image positions for a thin lens. RICHARD C. HITCHCOCK, *U. S. Naval Academy.*—A wooden model lens system uses dowels for light rays and for the principal axis. Three sets of objects and corresponding images give typical solutions of the thin-lens formula, $1/f=(1/p)+(1/q)$. A pivoted brass rod, representing a light ray, is rotated to line up the objects, successively, with images that are (i) real and inverted, (ii) virtual and erect, and (iii) real and erect. All objects and images can be separated from, or attached to, the model. For simplicity a fixed-focus "lens" is used, and an "object" of constant size is moved toward or away from the lens; all its "images" lie along the dowel that (*a*) passes through the focus away from the light source, and (*b*) crosses the plane of the lens at a point spaced from the principal axis at a distance equal to the object size. Comparison of the models for a converging and for a diverging lens will show the mirror symmetry of the use of positive and negative values of f. This wooden lens system is visible over a wide angle of class view, and in a brightly lighted room.

14. Safeguards on laboratory apparatus. C. J. OVERBECK, *Northwestern University.*—It is neither possible nor desirable to safeguard laboratory apparatus completely. The laboratory must be a place where the student learns by doing; some mistakes, resulting from careless thinking or manipulation, are a part of the learning process. Safeguards should be provided when a particular type of error reduces the teaching effectiveness of the laboratory. Several simple safeguards, some doubtless also used elsewhere, have been found helpful in our first-year laboratory. These include a "frictionless" magnetic time recorder to replace the annoyance of high potential; the use of "open" snap switches for protecting electric devices such as heater units; nondrop weight hangers; protection for galvanometers and lenses.

15. Potential energy. F. W. WARBURTON, *University of Kentucky.*—The following definition is proposed: Potential energy is the work that may be done by the action of one body on another. It is distinct from kinetic energy of a body, which is the work done in changing the momentum of this one body. Potential energy is thus always a function of relative position, but not solely of position. Examples were given to show the utility of this definition in the classroom. Specific phrases such as "electric potential energy of the atoms in the spring" are preferred to the type, "energy stored in the spring." Elastic impact is described as transfer of kinetic energy of the approaching body to electric potential energy of the charges in the deformed bodies, thence transfer to the kinetic energy of the second body. By the unobtrusive mutual electric action, energy is conserved during the short interval of impact when the kinetic energy of both bodies is small. Potential energy includes magnetic potential energy of moving charges and avoids the anomaly of using work done *against* the magnetic force (overcoming back induced electric field) as the energy in the Lagrangian function from which the formula for magnetic force is found.

16. Can all physics experiments utilize graphical methods of analysis? LAWRENCE E. KINSLER, *U. S. Naval Academy.*—Students appear to derive the most knowledge from physics experiments whose analysis centers about a graph of the data. Such a pictoral representation seems to offer more of interest and understanding than does the mere substitution of numbers into equations. Graphical methods tend to emphasize: (i) the correlation between the graph of a mathematical equation and the graph of physical data; (ii) methods of properly weighting individual measurements in determining a final result; (iii) understanding of the physics involved rather than a mere search for a correct answer; (iv) training in graphical methods, which is of particular importance to engineering students. It has been found most profitable to use graphical methods whenever the data may be plotted so as to produce a straight line. The determination of the moment of inertia of a small gyroscope rotor offers an excellent illustration. If a series of applied torques is plotted as ordinates, and the corresponding angular accelerations as abscissas, the slope of the resulting line gives the moment of inertia of the rotor. Furthermore, by extrapolating this line the student may be led to draw significant conclusions concerning its intercepts on the two axes. A comprehensive list of suggested experiments using graphical methods of analysis was presented.

17. Challenging problems for general physics classes. LOUIS R. WEBER, *Colorado A. & M. College.*—The lecturer may fulfill J. J. Thomson's hope that the proper function of lectures is not to give a student all the information he needs, but to rouse his enthusiasm so that he will gather knowledge himself, perhaps under difficulties.[1] Generally, however, the problems assigned to students merely are numbers on a list, having none of the flavor of the personality of the lecturer. This objectionable feature may be eliminated by using the following technic. Assume that Archimedes' principle as applied to gases is being discussed. Soap bubbles are blown with illuminating gas, some rising and some falling. One bubble neither rises nor falls. A student finds the approximate diameter with a ruler. The class is then asked to compute the thickness of the bubble wall. During the lecture, several excellent problems of this kind are always born. The solutions, which are brought to class the next period, have required certain assumptions and the application of specific knowledge (which invariably takes the student to the library). Students generally enjoy such problems; for one thing, they realize that the instructor himself is interested in the solution.

[1] Lord Rayleigh, *The life of Sir J. J. Thomson* (Cambridge Univ. Press, 1943), p. 241.

18. Some demonstrations in intermediate mechanics. MARIO IONA, JR., *University of Chicago.*—Three demonstrations were described which should be helpful in clarifying relations in mechanics that are usually not discussed in beginning courses because they are regarded as too difficult. (i) A simple generalization of the compound pendulum illustrates that the period depends only on the orientation of the axis and its distance from the center of mass, but not on the position of the axis. (ii) The path of a rolling ball with reference to a coordinate system at rest is observed by means of shadow projection, and, at the same time, in a rotating coordinate system, by allowing the ball to mark its path; the curved path in the rotating system demonstrates the necessity of introducing the Coriolis force. (iii) By demonstrating that a rigid body can rotate independently about an axis through the center of mass, even if the whole object is turned around a fixed axis, one can show the significance of knowing the moment of inertia with respect to an axis through the center of mass if relations for the moments of inertia with respect to parallel axes are desired (Steiner's theorem).

19. Parallels between physics and stellar dynamics. HERBERT JEHLE, *Harvard University.*—The simplest approach to stellar dynamics has been the one that neglects interactions between nearby stars and takes into consideration only the smooth field of force which comes from a smeared-out mass distribution of the whole stellar system. This is, however, a semi-empirical method because it is confined to systems in a state in which encounters may be neglected. As to the general problem, the interaction terms in astronomical systems come from very small long range forces caused by nearby stars, in contrast to the strong short range forces acting during a collision of molecules. Therefore, it is advisable to describe those interaction terms by a force fluctuation method, as in the theory of Brownian motion. The deviations of the actual field of force from the smooth average field of force are described statistically; they cause a slow convection and diffusion of the stars streaming in the six-dimensional phase space. The force fluctuation method, however, has its limitations in that it involves many specific assumptions about masses, velocities and distances between stars, and the question arises what other kind of statistical hypothesis is more adequate to the problem.

Attendance

The following members registered at the meeting:

E. F. Allen, Academy of the New Church; R. Andrews, Eastern High School (Washington, D. C.); Gladys Anslow, Smith College; Alice Armstrong, Wellesley College; R. H. Bacon, Fairchild Camera and Instrument Co.; C. P. Bailey, Bates College; E. S. Barr, Tulane University; B. W. Bartlett, U. S. Military Academy; H. A. Barton, American Institute of Physics; P. F. Bartunek, National Bureau of Standards; J. W. Beams, University of Virginia; C. E. Bennett, University of Maine; D. K. Berkey, Colgate University; W. H. Billhartz, Franklin College; H. Louisa Billings, Smith College; L. I. Bockstahler, Northwestern University; R. A. Boyer, Muhlenberg College; T. B. Brown, George Washington University; P. J. Burke, American Council on Education; G. H. Burnham, Norwich University; S. M. Christian, Agnes Scott College; M. A. Countryman, Illinois Institute of Technology; S. W. Cram, Kansas State Teachers College; H. L. Dodge, Norwich University; Clare Driscoll, Veterans High School Center (Washington, D. C.); J. B. Davis, Lower Merion Senior High School (Ardmore, Pa.); F. G. Dunnington, Rutgers University; Barbara Dwight, Massachusetts Institute of Technology; V. E. Eaton, Wesleyan University; F. K. Elder, U. S. Navy; J. D. Elder, Wabash College; I. Estermann, Carnegie Institute of Technology; F. M. Exner, Minneapolis-Honeywell Regulator Co.; Goldena Farnsworth, Hollins College; C. H. Fay, Shell Oil Co.; Ellinor B. Fox, Vassar College; I. M. Freeman, Swarthmore College; G. C. Fromm, Capital University; R. C. Gibbs,

Cornell University; L. R. Hafstad, Carnegie Institution of Washington; A. W. Hanson, The Citadel; W. H. Hartwell, University of New Hampshire; S. M. Heflin, Virginia Military Institute; C. L. Henshaw, Colgate University; Erna M. J. Herry, Queens College; R. C. Hitchcock, U. S. Naval Academy; F. F. Householder, University of Akron; L. G. Hoxton, University of Virginia; H. H. Hubbell, Jr., Princeton University; G. F. Hull, Jr., Dartmouth College; J. M. Hunter, Virginia State College; J. M. Hyatt, Simmons College; M. Iona, Jr., University of Chicago; W. J. Jackson, Rutgers University; J. C. Jensen, Nebraska Wesleyan University; R. N. Jones, Philadelphia College of Pharmacy and Science; G. E. C. Kauffman, University of Delaware; J. M. Kelley, Loyola High School (Baltimore, Md.); W. E. Kelly, University of Pittsburgh; L. E. Kinsler, U. S. Naval Academy; D. E. Kirkpatrick, Queens College; P. E. Klopsteg, Northwestern University; R. T. Lagemann, Emory University; L. C. Langguth, Fairfield College Preparatory School; H. B. Lemon, University of Chicago; R. B. Lindsay, Brown University; E. M. Little, Aberdeen Proving Ground; A. Longacre, Radiation Laboratory (Cambridge, Mass.); Lilly Lorentz, Smith College; F. E. Lowance, Georgia School of Technology; W. N. Lowry, Bucknell University; A. L. Lutz, Wittenberg College; J. Lynch, Fordham University; K. V. Manning, Pennsylvania State College; A. B. Meservey, Dartmouth College; Helen Messenger, Hunter College; C. W. Miller, Michigan State College; J. Mokre, Barat College; R. B. Morrissey, Manhattanville College; B. M. Leighton, City College (New York, N. Y.); C. C. Murdock, Cornell University; W. W. Mutch, Knox College; R. D. Meyers, University of Maryland; Louise S. McDowell, Wellesley College; S. H. McIntire, Norwich University; D. R. McMillan, Emory University; J. H. McMillen, Princeton University; W. Noll, Berea College; H. N. Otis, Hunter College; C. J. Overbeck, Northwestern University; R. F. Paton, University of Illinois; R. A. Patterson, Rensselaer Polytechnic Institute; H. A. Perkins, Trinity College; E. R. Pinkston, U. S. Naval Academy; M. L. Pool, Ohio State University; D. N. Read, Cooper Union; W. H. Robinson, North Carolina College for Negroes; E. M. Rogers, Princeton University; D. Roller, Wabash College; P. Rood, Western Michigan College; Y. K. Roots, Western College; P. Rosenberg, Paul Rosenberg Associates (New York, N. Y.); D. Ross, Pennsylvania State College; W. H. Ross, Massachusetts State College; A. E. Ruark, Naval Research Laboratory; J. H. Rush, Clinton Laboratories; R. D. Rusk, Mt. Holyoke College; E. A. Schuchard, Naval Ordnance Laboratory; M. W. Schwinn, Oberlin College; F. Seitz, Carnegie Institute of Technology; T. C. Sermon, Michigan College of Mining and Technology; R. S. Shankland, Case School of Applied Science; R. S. Shaw, City College (New York, N. Y.); F. G. Slack, Vanderbilt University; R. D. Spangler, Du Pont de Nemours & Co.; L. P. Smith, RCA Laboratories; D. W. Steinhaus, Edgewood Arsenal; Hildegard Sticklen, Sweet Briar College; R. M. Sutton, Haverford College; L. W. Taylor, Oberlin College; E. W. Thatcher, Union College; E. W. Thomson, U. S. Naval Academy; J. A. Tobin, Boston College; Agnes Townsend, Barnard College; M. H. Trytten, National Research Council; K. S. Van Dyke, Wesleyan University; Katherine M. Van Horn, New Jersey College for Women; F. L. Verwiebe, Johns Hopkins University; F. W. Warburton, University of Kentucky; B. B. Watson, University of Pennsylvania; W. Webb, Pennsylvania State College; R. C. Weaver, Virginia Military Institute; Dorothy Weeks, Wilson College; M. R. Wehr, Swarthmore College; M. W. White, Pennsylvania State College; S. R. Williams, Amherst College; J. G. Winans, University of Wisconsin; B. F. Wissler, Middlebury College; K. S. Woodcock, Bates College; R. M. Woods, University of Chattanooga; Rosalyn S. Yalow, Federal Telecommunications Laboratory; V. J. Young, Sperry Gyroscope Co.; I. F. Zartman, Johns Hopkins University; M. W. Zemansky, City College (New York, N. Y.).

Report of the Secretary

The Executive Committee of the American Association of Physics Teachers held its annual meeting in the American Institute of Physics building, New York City, on January 24, 1946. President R. C. Gibbs presided. The committee was in session from 7:30 P.M. to 12:15 A.M.

Members of the Association who attended were: *L. I. Bockstahler, J. C. Boyce, *T. B. Brown, M. A. Countryman, H. L. Dodge, J. D. Elder, *R. C. Gibbs, C. J. Lapp, *K. Lark-Horovitz, R. E. Lowance, *W. Noll, *T. H. Osgood, *C. J. Overbeck, *R. F. Paton, *W. B. Pietenpol, *D. Roller, *A. E. Ruark, M. N. States, R. M. Sutton, *L. W. Taylor, B. B. Watson, *M. R. Wehr, *M. W. White. Asterisks indicate the names of those serving on the executive committee for this meeting. The others were present by invitation.

Business with American Institute of Physics.—A letter dated November 14, 1945 from its secretary stated that the Governing Board of the Institute had adopted the following resolution: "That the 1946 contributions of the Member Societies in support of the Institute be fixed at 15 percent of the dues collected from individual members in 1945." This is in accordance with the plan adopted in 1945.

H. A. Barton, Director of the Institute, reported on various activities of the Institute.

R. C. Gibbs was elected to serve, with P. E. Klopsteg and L. W. Taylor, as representative of the Association on the Governing Board of the Institute for a three-year term.

Reports of officers.—Each officer made a brief report of the work for the year. The annual report of the Treasurer appears elsewhere in this issue.

The Journal.—Upon the recommendation of the editor, A. L. Hughes and M. Zemansky were appointed associate editors for the three-year period, 1946–1949. The editor has been authorized to select and appoint a third associate editor to serve during the same period. The journal committee is making a study of the advisability of increasing the number of issues from six to perhaps nine per year. A final report from this committee will be forthcoming.

Committee on membership.—L. W. Taylor reported on the activity and success of this committee. The total membership has increased from 1140 as of January 1945 to about 1350 as of January 1946.

Budget committee.—Upon the recommendation of this committee, as presented by L. W. Taylor, the sum of $1200 was set aside for office and traveling expenses of the editor, and $300 as partial compensation for the assistant editor.

The Secretary's office was allocated $600 for office supplies and help, printing of notices and programs, etc. It was voted "to pay the secretary's traveling expenses to regular meetings of the Association at which the secretary must be present."

The executive committee voted to hold $5000 in bonds as a reserve fund for Journal use. A sum of about $440 was authorized to help defray expenses associated with the work of the committees on membership and on the teaching of physics in secondary schools, and of the Association representatives on the American Council on Education.

Policy committee.—To correct overlapping of committee work and to coordinate our activities, a policy committee

has been instituted. It will survey the work of existing committees and make recommendations.

Other committees.—Written or brief oral reports were accepted from the chairmen of other committees. Some of these reports will appear in print.

Regional chapters.—All ten chapters of the Association have filed their annual reports with the secretary. Additional chapters are in process of organization.

Future meetings.—The next meeting of the Association will be in cooperation with the Society for the Promotion of Engineering Education, at Washington University, St. Louis, Missouri, during the week of June 17.

The material presented at the New York meeting is again available for use at local meetings. Arrangements may be made by writing to Dr. L. W. Taylor, Oberlin College, Oberlin, Ohio.

A new directory of our membership will be issued by the secretary later this year. In order to have it as up-to-date as possible, members of the Association are urged to report any contemplated change of address as soon as possible.

Annual business meeting.—The fifteenth annual business meeting was held on January 26, 1946, at 11:30 A.M. The secretary reported on the work of the Association and its executive committee. The report of the tellers of the annual election was read; the 522 ballots counted resulted in the election of the following:

President: R. C. Gibbs.
Vice President: Paul Kirkpatrick.
Members of Executive Committee: R. B. Lindsay; J. D. Stranathan.

President Gibbs spoke on the question of a contribution to the American Institute of Physics building fund. All member societies, except the Association, have made contributions to this fund. The motion, from the floor, that we make a $1000 contribution, passed unanimously.

C. J. OVERBECK, *Secretary*

Annual Report of the Treasurer

Balance brought forward from Dec. 15, 1944		$ 3927.21
CASH RECEIVED		
Dues received from members	$5620.00	
Dues received for 1943	5.00	
Dues received for 1944	47.50	
Dues received for 1946	397.50	
Dues received for 1947	5.00	
Royalties, *Demonstration experiments in physics*	112.78	
Constituent Membership in ACE, paid by American Institute of Physics	100.00	
Bond coupons	125.39	
Total deposited, 12/15/44 to 12/15/45		6413.17
Total cash available		$10340.38

DISBURSEMENTS		
Postage and supplies	$ 215.50	
Printing	280.60	
Secretary's office expense	96.53	
Constituent membership in ACE	100.00	
Stenographer, editor's office	679.20	
Editors' traveling expense	313.63	
Payments to American Institute of Physics	3393.52	
Miscellaneous traveling expense	68.09	
Discount on checks	11.94	
Checks for dues returned	20.00	
Miscellaneous	33.65	
Total disbursed		5212.66
Cash on deposit Dec. 15, 1945[1]		$ 5127.72

PAUL E. KLOPSTEG, *Treasurer*

I have audited the books of account and records of Dr. Paul E. Klopsteg, Treasurer of the American Association of Physics Teachers, for the year ended December 15, 1945, and hereby certify that the foregoing statement of receipts and disbursements correctly reflects the information contained in the books of account. Receipts during the year were satisfactorily reconciled with deposits as shown on the bank statements, and all disbursements have been satisfactorily supported by vouchers or other documentary evidence.

U. S. Government Bonds and Notes of $5000 par value were examined and the interest therefrom was satisfactorily accounted for.

WILLIAM J. LUBY
Certified Public Accountant

Evanston, Illinois,
January 22, 1946.

[1] Approximately $1000 is still due the American Institute of Physics for the publication of the journal for 1945. In addition to the balance indicated, the Association holds U. S. Government Treasury Bonds and Notes of par value $5000.

DIGEST OF PERIODICAL LITERATURE

Collection of Spilled Mercury

A piece of stiff silver foil, in the shape of a pointed spoon $\frac{1}{4}$ in. wide, is inserted into any convenient handle. A dime can be used if necessary. The silver is rubbed with a little mercury to form a bright layer of amalgam. The amalgam surface is readily wetted by mercury and a droplet of it is drawn onto the spoon without any effort of scooping. When a fairly large drop has been collected, it is shaken off into a bottle. A. F. McGuinn, *J. Chem. Ed.* **22**, 463 (1945).

Variation on an Old Experiment in Electrostatics

Lord Rayleigh has suggested the following modification of an experiment first performed by Newton. A small piece of very thin aluminum foil is cut in the shape of an isoceles triangle with rather small vertex angle. A notch is cut from the base, and the two sides are bent at right angles to the plane of the triangle but in opposite directions. This piece of foil is laid on a sheet of bright tin, 4 cm above which is a copper hemisphere 13 cm in diameter, mounted on an insulating support. The bowl is charged positively with respect to the tin sheet by connecting bowl and sheet to the terminals of a Wimshurst machine. When the machine is worked, the bit of aluminum foil rises onto its point and spins rapidly. If the potential difference is increased, the spinner rises into the air and remains suspended, rotating rapidly all the time. The effect is due to the mechanical reactions of the electric discharges from the ends of the two bent legs and the point. The discharges can be seen if the experiment is carried out in the dark.—*Sch. Sci. Rev.* **26**, 214–215 (1945); quoted from *Mo. Sci. News* (Sept. 1944).

Internal Resistance of a Daniell Cell

A common elementary experiment is the measurement of the internal resistance r of a cell, defined by the equation $r=(E-v)/i$, where E is the emf of the cell, v its terminal potential difference and i the current. If this equation is put in the form $v=E-ri$, then r may be obtained from the slope of the graph of v as a function of i, since it is generally assumed that E is constant so that the graph is a straight line.

The graph can be extended in both directions by using storage cells aiding or opposing the cell under study. If this is done with a fresh zinc sulfate Daniell cell, it is found that the graph is not a straight line, although it asymptotically approaches two parallel straight lines on either side of the point for zero current. Similar experiments have been conducted with alternating current in the cell.

The results of these experiments lead to the following conclusions: (i) neither the emf nor the resistance vary with the current at the moment; (ii) the emf of a Daniell cell depends on the past history of the cell; (iii) the internal resistance of a Daniell cell is almost unaffected by the past history of the cell.—C. Ashford, *Sch. Sci. Rev.* **25**, 42–48 (1943).

Physics-Chemistry Sequence

Overlapping of the fields of physics and chemistry leads to the inclusion of many common topics in first-year courses in both subjects. Such topics as fundamental units, density, pressure, the gas laws, kinetic theory, spectra, electrolysis, radioactivity, atomic and nuclear structure are frequently taught twice, with resultant loss of time. In a liberal arts college, chemistry should probably be taught first, because it is essentially non-mathematical. In a technical institution, where all students begin mathematics on entrance and are required to take both physics and chemistry, physics should come first. Thus a foundation may be laid on which purely chemical instruction may be based. If the physics and chemistry departments are not separated, so that there is no rivalry between them, the order of presentation of topics can be chosen without regard to whether they are "really" physics or chemistry. In other institutions, cooperation between the separate departments could lead to economy in time and hence the presentation of more material. J. B. Hoag, *J. Chem. Ed.* **22**, 152–154 (1945).

Social Relations of Science

Before the first World War science was generally a small-scale activity, pursued by university professors, wealthy amateurs and private industrial firms. After that war it was realized that research must be pursued systematically, and this led to the creation of research bodies of various forms. The present war will lead to changes as great as those after the last war. The need for scientists is great and must be met by education. There must also be provision for using second- and third-rate persons where they can do useful work under direction, rather than relying solely on first-rate minds to carry out the research. A new kind of science education will be necessary that will include the study of the relations of science to other human activities. The foundation for these studies will be the history of science. Such topics as the influence of science on literature will aid in stimulating interest in literature. The influence of science on history, past and present, should interest the scientist in history. An essential new profession is that of science public relations officer, to interpret science to the people who pay for it.

The primary problem now is the simultaneous promotion and assimilation of science. Intellectual curiosity is not enough; the social belief that science is a worthy pursuit is also necessary. So are freedom of thought, of speech and of action. But the condition for freedom is discipline, which is why planning is now essential. It is the province of the science teacher to take the lead, and to inform the statesmen what must be done. Improvement in the technic of teaching is no longer enough. We must all be statesmen as well as teachers. J. G. Crowther, *Sch. Sci. Rev.* **26**, 268–284 (1945).